BENEFITS AND PITFALLS OF NETWORK INTERCONNECTION[*]

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Abstract

This paper assesses the private and social incentives for disjoint networks to interconnect under various ownership structures. Terms of interconnection are derived for a noncooperative equilibrium. We find that networks mutually profit from interconnection when it creates new services that did not exist beforehand, but also when it creates services that compete directly with existing ones. Given the opportunity to move first, an integrated network will choose not to foreclose its non-integrated rivals. Generally we find that when two or more networks contribute components to a service, double marginalization reduces industry profit and consumer surplus. For this reason, divestiture often harms consumers as well as lowering network profits. Competitive supply of gateway services reduces profit and surplus, but individual networks profit by selling off these facilities to a third party. In contrast, an integrated network will not voluntarily divest its end-to-end service. Compulsory divestiture may inflict serious harm, not only on owners of the integrated network, but on consumers as well.

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1. INTRODUCTION

Network services are created by packaging separate components in combinations demanded by users. Typically, production of individual components exhibit strong scale economies, and so their supply tends to be monopolized. Consequently, unless a single firm provides all components, disjoint networks must interconnect to ensure users a full array of services.[1] From users' perspective, interconnection increases the variety of services from which they can choose. Network operators are concerned with the fact that these services may compete horizontally or vertically with existing products. Our primary goal is to assess the private and social incentives for rival networks to interconnect so as to expand the range of services.

We start with an integrated network that can provide end-to-end service without need for interconnection. A competing non-integrated network produces *only one* component that, when combined with a component of the integrated network, creates an imperfect substitute for end-to-end service. We analyze pricing in this setting, and in a number of other structures that are progressively more decentralized. We also evaluate the incentives of each network to divest or acquire specific vertically- or horizontally-related links.

The basic structures of our paper appear repeatedly in the telecommunications industry. Cities are served by single local exchange networks which, in turn, are connected by a nationwide inter-exchange network. After competition broke out in the U.S. in the 1970s, the new long distance companies were force to seek interconnection with local telephone companies, most of which were owned by their chief competitor AT&T. They paid "access charges" for use of local facilities that originate and terminate long-distance calls. Initially, the new inter-exchange carriers negotiated discounts to compensate for connections that were technically inferior to those available to AT&T.[2] Divestiture separated AT&T's local and long distance networks, and the

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[1] Bittlingmayer (1989), Sharkey (1991) and Woroch (1990) show that, even under plausible assumptions on technology, the cost-minimizing network will fragment under the pressure of competition.

[2] Exchange Network Facilities and Interconnection Agreements, 71 FCC 2d 440, 1979.

Modified Final Judgment mandated non-discriminatory access for all inter-exchange carriers.[3] The level and structure of access charges have evolved through a series of Federal Communications Commission orders.[4]

Information providers--such as voice mail services--also need interconnection with local networks. When the local exchange carrier provides a similar service, it was required to form a structurally separate subsidiary.[5] These requirements were later replaced by non-structural safeguards, which amounted to accounting rules governing transfer prices.[6] Under the concept of "open network architecture," they would supply information providers interconnection under the same technical and monetary terms as the local network's information service.[7]

A few years ago, so-called *competitive access providers*, or CAPs for short, sprang up to offer high capacity services in competition with local telephone company offerings. A major source of CAP revenues is selling circuits that connect large businesses to their preferred long distance carriers. This is an example of "complete bypass" of local exchange facilities. Recently, the CAPs won permission from the FCC to "collocate" their lines and equipment in LECs' central offices.[8] This allows them to access traffic gathered by the local company without the cost of duplicating its entire network.

While most of our examples are drawn from telecommunications, interconnection is critical for efficient operation of many other network industries. The electric power industry is a good example. In the early days of this industry, two standards of power transmission

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[3] AT&T v. United States, 552 F. Supp. 133, 1982, Modification of Final Judgement.

[4] MTS and WATS Market Structure, Amendment of Part 67 of the Commission's Rules, CC Docket 78-72. Access charges total nearly one half of AT&T's operating expenses.

[5] FCC Report and Order, In the Matter of Procedures for Implementing Detariffing of Customer Premises Equipment and Enhanced Services (Computer Inquiry II), 95 FCC 2d 1276.

[6] FCC Report and Order, Amendment of Sections 64.702 of the Commissions Rules and Regulations (Computer Inquiry III), CC Docket 86-252 (released June 1986).

[7] Filing and Review of Open Network Architecture Plans, 4 FCC Rcd 1 (1988), and FCC's Docket 88-2, Phase I, FCC 90-134 (released May 8, 1990).

[8] Expanded interconnection with local telephone company facilities, Notice of Proposed Rulemaking and Notice of Inquiry, CC Docket 91-141, 6 FCC Rcd 3259, June 1991.

coexisted on disjoint networks.[9] The direct current standard was adopted in urban distribution systems, accounting for about two-thirds of installed generating capacity. The remainder used the alternating current method that enjoyed a cost advantage in serving the rural and outlying areas. Interconnection between the two systems awaited the development of the rotary converter. It gave powerplants expanded access to users and also added to the ranks of generators that could deliver power to the system.[10]

The transportation industry has elaborate arrangements to convey shipments across non-overlapping networks. In the early days, this involved loading and unloading of freight in when lines used different track gauges. After standardization of track and equipment, interlining agreements allowed rolling stock to travel over contiguous rail networks without transferring shipments. The trucking and rail industries have also interconnected to offer shippers end-to-end service. Under the "piggyback system," tractor trailers are loaded aboard flatbed rail cars for the rail portion of their journey.

The importance of interconnection is by no means limited to industries that deliver service over a physical network. In high technology industries that have adopted an *open architecture framework* and are ruled by certain *compatibility standards*, it is common to have various firms sell component parts of a complete system -- such as computer, audio or video systems. Often a vertically-integrated firm will produce all components, i.e., a *full system*, while others specialize in one or more of the component parts. For example, traditionally computer manufacturers produced both hardware and software, and this remains true for mainframe computers to this day. However, in the personal computer market typically hardware and software are sold by separate

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[9] See David and Bunn (1988).

[10] The increasing scales of central station generation lead to rapid concentration in supply, the industry became dominated by large, vertically-integrated, investor-owned utilities. Nearly a third of their production was sold to smaller, non-integrated or partially integrated utilities. Very often, third parties would undertake the job of transmitting or "wheeling" power between the two utilities. During the 1970s plant scales exhausted increasing returns. At the same time the industry met with rising fuel prices and increased environmental and safety concerns. The Public Utility Regulatory Policy Act of 1978 sought to foster the re-emergence of non-utility generators, among other goals. The NUGs are allowed to sell back power at rates not to exceed the avoided cost faced by the local company. See Joskow (1989).

firms.[11] Foreclosure and vertical price squeeze have been key policy issues surrounding the development of these industries.

Other relevant issues discussed in this paper include the effects of increasing competition in the market for one component on the overall market and the effects of vertical fragmentation of a full system manufacturer in comparison with vertical fragmentation of a provider of components that cannot provide full system (end-to-end) service.

2. A MODEL OF INTERMODAL COMPETITION

We discuss network interconnection and competition in the context of a simple network. There are two *end-nodes* A and B, plus two *intermediate nodes* S and T that we call *switches*. A *link* or *component* connects a pair of nodes of either type. Four components are possible in our setting: AS, SB, ST and BT. Components can be combined in various ways to complete the service between A and B. In Figure 1, observe that the two routes between A and B pass through either or both of the switches at S and T. The link ST that connects the two switches is a special component called the *gateway*.[12] *Indirect service* between A and B includes the gateway ST in addition to links AS and BT. *Direct service* uses only AS and BS. If two components are supplied by different firms, a service is called a *hybrid*. Note that direct service can be a hybrid if one network supplies AS and another supplies SB.

<< Insert Figure 1 >>

Users derive utility from service between A and B. They place no value on individual components themselves, so they only have a derived demand for the vertically related components. Users do not experience any disutility from hybrids compared to service provided by a single firm. Finally, we assume that users view direct and indirect services as imperfect substitutes.

Five different ownership structures are displayed in Figure 1. In all cases, network 1 holds a monopoly over link AS; ownership of other links varies. Case 1 is our reference

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[11] Apple, which sells both the Macintosh computer and its basic operating system, is an exception. Nevertheless, software applications written for this platform are available from dozens of independent firms.

[12] If we differentiated network services by the direction of traffic (as is necessary in transportation models), then we would have to decide whether the gateway offered uni-directional or bi-directional interconnection service.

ownership structure. We name it *intermodal competition*. Network 1 provides end-to-end service ASB, while network 2 provides only the part STB of indirect service ASTB.

This ownership structure parallels the situation that existed after competition in the long distance service began in the U.S., but before AT&T was divested. The end-link AS can be thought of as a Bell operating company's "local loop" that connects a customer A to a local switch S. This customer could complete a long distance call to user B in another city over AT&T's Long Lines network SB. Alternatively, he could choose a competitor such as MCI or Sprint. In that case, the call would travel from the local switch to an "access tandem," T, and from there on the competitive carrier's inter-city network BT.

Case 1 also represents a structure that is emerging with the appearance of competitive access providers. In that case, the CAP builds an optical fiber network that connects two locations of the same firm within a city, A and B, such as branch offices of a bank. The company can transmit voice and data between the two offices over the public switched network ASB. Alternatively, it can rely on a CAP to supply part (or all) of the service. In case 1, the CAP's network simply connects office B to the local switch, bypassing a portion of the local network.[13]

In the personal computer industry, case 1 can represent the market for word-processing services for disk operating systems (DOS). Microsoft's *MS-DOS* has the virtual monopoly of the DOS operating systems (link AS). Microsoft also publishes *Microsoft Word*, a popular word processor (link SB). It faces stiff competition in the word-processing market by *Wordperfect* (link STB).

The integrated network has good reasons to refuse to interconnect with a rival. Clearly the non-integrated network cannot survive without interconnection. In addition, interconnection allows the competitor to steal business from its monopolized service. In fact, we find that <u>it will not foreclose the rival network</u>. The reason is that, having kept its rival out, the integrated network stands to earn monopoly profits *only* on its end-to-end service. It forgoes additional

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[13] Complete bypass would occur if it also provided link AT so that it could offer end-to-end service ATB in direct competition with the local carrier. If AT were provided, the model would coincide with the framework of Economides and Salop (1992) and Economides (1991) where there are two substitute components of each type. In the terminology of these papers the components are identified as follows, AS $\rightarrow A_1$, AT $\rightarrow A_2$, BS $\rightarrow B_1$, and BT $\rightarrow B_2$. The price of ST is identified in Economides (1991) with the cost R of the adapter that ensures compatibility between the components.

profits from the hybrid service which are non-negligible since it monopolizes an essential component.

While the substitute service generates profits for both networks, its creation is a mixed blessing for consumers as a group. The reason is that price is actually *lower* if a single, integrated firm provides the complete service rather than two interconnected networks. Competition raises price because each non-integrated network adds a markup to the other network's price. This observation can be traced back to the work of Cournot (1838).[14] Spengler (1950) coined the term "double marginalization" for this phenomenon.[15]

Interconnection also opens the possibility of price discrimination by a network holding a monopoly over some component. A long-standing fear of network regulators is the possibility that an integrated network will execute a vertical price squeeze by selling its component service to its own subsidiary at advantageous rates compared with rates charged to a competitor. Returning to the original case, we observe the integrated network engaging in price discrimination in the form of mixed bundling. Here, the price of end-to-end service can fall *below* the price of just one of its components as long as demand for the hybrid is sufficiently inelastic.[16]

Case 2 represents *perfect collusion*, or equivalently, ownership of all links by the same network. AT&T's infamous end-to-end monopoly prior to inter-exchange competition and prior to the 1984 divestiture exemplifies this structure, which is represented by Case 2 in Figure 1. The alternative service STB available to a customer at B can be thought of as cellular mobile phone service. It supplements traditional landline service represented by ASB. In many countries the national telecommunications provider monopolizes both services.

Case 3 introduces *competitive interconnection*. This case results from further fragmentation of the structure of intermodal competition (Case 1). Specifically, the gateway ST has been sold off to a third party. The resulting structure looks much like one envisioned for the

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[14] It has reappeared recently in a network (or compatibility) context in Baumol (1983), Sharkey (1987), Matutes and Regibeau (1988), Economides (1989, 1991), and Economides and Salop (1992).

[15] To be precise, Spengler (1950) showed double marginalization in a model of *sequential* choice of prices of complementary goods. Cournot (1838) showed the same result in a model of *simultaneous* choice. Sonnenschein (1968) showed the formal equivalence between Cournot's model of price-setting firms that produce complements with the more well known model of the same author where quantity-setting firms produce substitutes.

[16] It should be remembered that such an occurrence is not necessarily an indication of anticompetitive behavior: Ramsey optimum structure requires this general property of the price structure.

future public network following deployment of "personal communications services." So-called PCS would equip customers with the equivalent of a go-anywhere cordless phone. Then a customer like B would have the option of using the mobile phone when away from home or office. Under some proposals, an independent wireless network would provide a radio-link BT from the customer to a "mobile telephone switching office" at T. To reach other customers who only have stationary phones, such as A, the wireless network must interconnect with the landline system at the switch S. Various entities are in a position to provide interconnection service ST, including existing cellular operators, competitive access providers, and even cable TV companies.

A question arises naturally as we consider different ownership structures: When would one of the incumbent networks choose to divest itself of a component? To begin with, suppose one of the two original networks owned the gateway that interconnects them. We see that each of the two networks would prefer that the network own the gateway because ownership does not generate any revenues, yet there is some fixed cost of building and operating it.

Nevertheless, given the option, either network could profit by selling the gateway to a *third party,* thus resulting in case 3. Such divestiture would lead to prices affected by *triple* marginalization, which reduces profits of the original two networks, as well as overall industry profits. However, the original owner of the gateway can be better off if it sells this asset at a price equal to the full post-divestiture profits of a third party. Of course, this is only possible because the divesting network takes advantage of the weaker post-divestiture competitive position of the other original network. Importantly, prices for the end-to-end service also rise, clearly reducing consumer surplus compared to the situation when the original networks own the gateway--either individually or jointly.

In case 4, we examine what might be called *independent ownership.* Compared to Case 1, the AS and SB links are now placed under separate ownership. This represents the arrangement that existed after AT&T’s divestiture and prior to complete implementation of "equal access." In that case, AT&T Communications would require access to local networks AS alongside the other major inter-exchange carriers. Here the link AS can either be the spun-off Baby Bell companies or an Independent. Ownership of ST by competitive carriers is an attempt to capture their unequal access to local switches.

We find that under no circumstances would the integrated network wish to divest routes such as SB that compete horizontally with its rival network. If forced to do so, prices of direct service rise because of double marginalization. Overall consumer welfare is harmed as well.

In Case 5, we discuss *independent ownership with competitive interconnection*. This case results from independent ownership (Case 3) through divestiture of the gateway ST. A good example of this ownership structure occurs in the case of competitive supply of high-capacity access. To this day, the majority of a CAP’s revenues derive from hauling long-distance traffic from one inter-exchange carrier to another. Imagine that S and T represent "points of presence" of two competing long distance carriers in a particular central city. Then some long distance calls between users A and B in distant cities may be transferred from one carrier to another, especially when one of them experiences a shortage of circuits. Acting as a so-called "carriers’ carrier," a competitive access provider establishes the gateway between the two.

3. NETWORK PRICING UNDER COMPETITION AND MONOPOLY

3.1 Case 1: Intermodal Competition, "i"

The first ownership structure has network 1 providing service ASB, and network 2 providing service STB. This is case 1 in Figure 1. We label this case "intermodal competition" and denote equilibrium values with the subscript "i." Note that network 1 has a monopoly on route AS, but faces competition in route ST. Network 2 will make no sales at all unless it is able to interconnect with Network 1. The following table summarizes the routes and the prices.

Service	**Price**	**Provider**
ASB (direct)	p_{ASB}	Network 1 (ASB)
AS (part of ASTB)	q_{AS}	Network 1 (ASB)
STB	q_{STB}	Network 2 (STB)
ASTB	$q_{AS} + q_{STB}$	Hybrid, Networks 1 & 2

Prices of components are denoted by q, while prices of composite services are denoted by p. Note that we allow price discrimination by network 1 so that it can charge a different price for link AS as part of ASB or ASTB. Let D_{ASB} be the quantity of direct service and D_{ASTB} be the quantity of indirect service, and assume a quadratic utility function,[17]

$$U(D_{ASB}, D_{ASTB}) = \alpha D_{ASB} + \alpha' D_{ASTB} - [\beta(D_{ASB})^2 + \beta'(D_{ASTB})^2 + 2\gamma D_{ASB}D_{ASTB}]/2.$$

The representative consumer maximizes $U(D_{ASB}, D_{ASTB}) - [p_{ASB}D_{ASB} + (q_{AS} + q_{STB})D_{ASTB}]$, resulting in linear inverse demands

$$p_{ASB} = \alpha - \beta D_{ASB} - \gamma D_{ASTB}, \quad q_{AS} + q_{STB} = \alpha' - \gamma D_{ASTB} - \beta' D_{ASB}.$$

$\gamma > 0$, since the two services are substitutes. Inverting this system,[18] we have demand curves that are linear in own prices and cross prices

$$D_{ASB} = a - bp_{ASB} + c(q_{AS} + q_{STB}),$$
$$D_{ASTB} = a' - b'(q_{AS} + q_{STB}) + cp_{ASB}.$$

We assume $a, a', b, b' > 0$.[19] Coefficient c (= $\gamma/(\beta\beta' - \gamma^2)$ in terms of the coefficients of the utility function) measures the cross-price effect. Since the services are substitutes, we assume $c > 0$, and larger values of c indicate closer substitutability between the services. Thus, we require $\beta\beta' > \gamma^2$ which implies $bb' > c^2$.[20]

In terms of costs, we assume a fixed cost for every link and zero marginal costs.[21] When the two networks are interconnected, their profits are given by

$$\Pi_{ASBi} = p_{ASB}D_{ASB} + q_{AS}D_{ASTB} - F_{ASB}, \quad \Pi_{STBi} = q_{STB}D_{ASTB} - F_{STB}.$$

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[17] We assume $\alpha, \alpha', \beta, \beta', \gamma > 0$.

[18] The two systems of equations are equivalent in the region of prices that results in positive demands $D_{ASB}, D_{ASTB} > 0$ provided that $\beta\beta' - \gamma^2 \neq 0$. The equivalent coefficients are $\alpha = (ab' + ca')/(bb' - c^2)$, $\alpha' = (a'b + ca)/(bb' - c^2)$, $\beta = b'/(bb' - c^2)$, $\beta' = b/(bb' - c^2)$, $\gamma = c/(bb' - c^2)$ or $a = (\alpha\beta' - \alpha'\gamma)/(\beta\beta' - \gamma^2)$, $a' = (\alpha'\beta - \alpha\gamma)/(\beta\beta' - \gamma^2)$, $b = \beta'/(\beta\beta' - \gamma^2)$, $b' = \beta/(\beta\beta' - \gamma^2)$, $c = \gamma/(\beta\beta' - \gamma^2)$. For details on invertability see Singh and Vives (1984).

[19] We require $\alpha\beta' > \alpha'\gamma$ and $\alpha'\beta > \alpha\gamma$ on the coefficients of the utility function so that $a, a' > 0$. $b, b' > 0$ are implied from our earlier assumptions on $\alpha, \alpha', \beta, \beta'$ and γ in the two previous footnotes.

[20] Occasionally we invoke the slightly more stringent set of conditions $b > c, b' > c$.

[21] We will use the notation F_{ASB} to mean $F_{AS} + F_{SB}$. There are no economies of scope. Also note that our model can easily accommodate constant positive marginal costs by re-interpreting price as a markup over marginal cost.

If the networks are not interconnected, they earn $\Pi_{ASBi} = p_{ASB}D_{ASB} - F_{ASB}$ and $\Pi_{STBi} = 0$.

Networks use prices as strategies.[22] Network 1 chooses p_{ASB} and q_{AS} given q_{STB}. Its best replies are given by

$$p^b_{ASB}(q_{STB}) = (ab' + a'c)/[2(bb' - c^2)],$$
$$q^b_{AS}(q_{STB}) = (a'b + ac)/[2(bb' - c^2)] - q_{STB}/2.$$

Network 2 chooses q_{STB} given p_{ASB} and q_{AS} together; its best reply is

$$q^b_{STB}(q_{AS}, p_{ASB}) = (a' + cp_{ASB})/(2b') - q_{AS}/2,$$

Observe that the only interaction occurs between prices q_{AS} and q_{STB}, and that p^b_{ASB} is independent of other prices. When firms choose prices simultaneously, the equilibrium prices are best replies to each other[23]

$$p_{ASBi} = (ab' + a'c)/[2(bb' - c^2)],$$
$$q_{ASi} = (2a'bb' + 3ab'c + a'c^2)/[6b'(bb' - c^2)], \quad q_{STBi} = a'/(3b').$$

The corresponding equilibrium profits are[24]

$$\Pi_{ASBi} = (4a'^2bb' + 9a^2b'^2 + 18aa'b'c + 5a'^2c^2)/[36b'(bb' - c^2)] - F_{ASB},$$
$$\Pi_{STBi} = a'^2/(9b') - F_{STB}.$$

Now Π_{ASBi} is greater than the profit network ASB could derive from satisfying demand solely for service ASB. And as long as F_{STB} is not too large, Π_{STBi} is greater than network STB could earn without interconnection, i.e., greater than zero. Thus, the networks *mutually* desire interconnection.

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[22] Throughout this paper we use uniform prices, mainly because we believe that in practice networks have difficulty in applying sorting and no-arbitrage required for efficient use of non-linear prices and two-part tariffs. For example, re-sellers can easily bring about arbitrage.

[23] Second-order conditions for profit maximization of network ASB require $bb' > c^2$, which has already been assumed.

[24] Consumers' and total surplus are

$$CS_i = (4a'^2bb' + 9a^2b'^2 + 18aa'b'c + 5a'^2c^2)/72b'(bb' - c^2),$$

$$W_i = (20a'^2bb' + 27a^2b'^2 + 54aa'b'c + 7a'^2c^2)/72b'(bb' - c^2) - F_{ASB} - F_{STB}.$$

We now compare the prices of services across networks at equilibrium. Of particular importance is the price that network ASB charges on route AS that allows access to it to network STB. Provided that $p_{ASB} \geq q_{AS}$, network ASB could post any combination of prices for routes AS and SB that add up to p_{ASB}. Thus, if pressed by regulatory rules on *non-discriminatory interconnection* that require it to charge the same price to competitors as it charges customers, network ASB could always decompose price p_{ASB} into q_{AS} for route AS and $p_{ASB} - q_{AS}$ for route SB. A violation of the restriction $p_{ASB} \geq q_{AS}$ can be interpreted as *pure bundling* where the non-integrated network is forced to buy end-to-end service ASB to obtain the service AS.[25]

What determines the pricing incentives of network ASB? Remember that network ASB has a monopoly on link AS and therefore its relative pricing of links depends on the differences in elasticity of demand across links. We have

$$p_{ASBi} - q_{ASi} = (-2a'bb' + 3ab'^2 - 3ab'c + 3a'b'c - a'c^2)/[6b'(bb' - c^2)],$$

As a benchmark, consider the case when the two demand functions, D_{ASB} and D_{ASTB} have the same price intercept: $a'/a = b'/b = z$. When $z = 1$, the demand functions for the final goods provided by the two routes coincide when prices are the same. As z gets small, demand D_{ASTB} pivots inward and becomes more inelastic in comparison with D_{ASB}. Substituting the symmetry condition into the price difference equations and rearranging yields

$$p_{ASBi} - q_{ASi} = az(-3bc - c^2 + b^2z + 3bcz) = az[3bc(z - 1) + (zb^2 - c^2)].$$

Clearly, at $z = 1$, this expression is positive. However, when $z < c^2/b^2 < 1$, it is negative and so $p_{ASBi} < q_{ASi}$, i.e., Network ASB charges Network STB a higher price for the link AS than it charges its customers for *end-to-end* service ASB. This is because, as z decreases, and the demand D_{ASTB} decreases, Network ASB takes advantage of the decreasing demand elasticity

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[25] This is reminiscent of price discrimination schemes of airlines, where the ticket for route ASB costs less than the ticket for route AS. It also is similar to a choice of "incompatibility" by a system manufacturer holding a monopoly over the technology of one component in a mix-and-match environment. An "incompatibility" (or pure bundling) strategy by this firm forces manufacturers of a single component to buy the whole system, disassemble it, and then use only one component. This was the situation faced by "clone" manufacturers of a portable version of the original Macintosh, who had to buy the Mac as a computer-and-monitor package, and then substitute in their LCD screen.

for the hybrid service. Thus, when the demand for the indirect route is very inelastic, network ASB will not be able to meet a regulatory test of non-discriminatory interconnection.

Proposition 1: Under intermodal competition, if the demands for direct and hybrid services are roughly equal, the price of direct service provided by the integrated network p_{ASB} is *higher* than q_{AS}. If the demand for hybrid services is relatively small and inelastic, the price for integrated service, p_{ASB}, can fall below the price charged to the rival network's customers for link AS, q_{AS}.

A more stringent regulatory requirement would have the price of service ASB above the combined prices of components of the hybrid service, i.e., $p_{ASB} \geq q_{AS} + q_{STB}$.[26] At the non-cooperative equilibrium, we find

$$\begin{aligned} p_{ASBi} - (q_{ASi} + q_{STBi}) &= (-4a'bb' + 3ab'^2 - 3ab'c + 3a'b'c + a'c^2)/[6b'(bb' - c^2)] \\ &= az(-3bc + c^2 - b^2z + 3bcz), \end{aligned}$$

and this is negative for equal size demands D_{ASB} and D_{ASTB}, i.e., at $z = 1$.[27] Thus, in this case, this regulatory requirement cannot be met.

The fact that we may have $p_{ASBi} < q_{ASi} + q_{STBi}$ is a direct result of *double marginalization*. Cournot (1838) showed that two monopolists selling complementary goods will charge *higher* prices in sum than a *single* vertically integrated monopolist selling both goods. Each independent monopolist faces a less elastic residual demand than the single integrated monopolist because equal changes in price result in higher changes in profits than for an integrated monopolist. Essentially, vertically-related firms underestimate the influence of their

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[26] A stronger test of discrimination would require the price of direct service should not be less than twice what is charged for a single component, i.e., $p_{ASB} \geq 2q_{AS}$.

$$p_{ASB} - 2q_{AS} = (-4a'bb' + 3ab'^2 - 6ab'c + 3a'b'c - 2a'c^2)/6b'(bb' - c^2).$$

At $a'/a = b'/b = 1$, $p_{ASB} - 2q_{AS} = -a(b^2 + 3bc - 2c^2)/6b(b^2 - c^2) < 0$. Thus, when the demand functions are equal, network ASB would not be able to meet this regulatory restriction.

[27] At $z = 1$, $p_{ASBi} - (q_{ASi} + q_{STBi}) = a(c^2 - b^2) < 0$.

price changes on the opponent's price.[28] Just as the standard Cournot duopoly results in higher output than monopoly, the *vertically* related duopoly results in a *higher price* than a vertically integrated monopoly.

Clearly, our two networks are vertically related since component AS (provided by Network ASB) is complementary to good STB (provided by Network STB). In addition, however, the two networks are also *horizontally* related, since they both provide service from S to B. Thus, besides double marginalization, there is a pure price discrimination effect. Network ASB has a monopoly on service AS, and there are two different goods, ASB and ASTB, each of which incorporate AS. Thus, *ceteris paribus*, network ASB will tend to charge a lower price for the good that has less elastic demand. This is a second order effect and it is always overwhelmed by the opposite double marginalization effect so that $p_{ASBi} < q_{ASi} + q_{STBi}$. The price discrimination effect shows up more in the comparison between p_{ASBi} and q_{ASi} where $p_{ASBi} > q_{ASi}$ when demand D_{ASTB} is very inelastic.

<u>Proposition 2</u>: The price of direct service provided by the integrated firm p_{ASB} is *lower* than the price of the competing indirect service when the demands for the two services are equal at equal prices. This result is reversed when the demand for indirect services is small and inelastic.[29]

Note that, as far as marginal decisions are concerned, it does not matter which of the two networks owns the interconnecting facility ST. As long as both networks are in operation, prices for services are identical if ST is owned by network 1 or network 2, or for that matter, they share the ownership through a joint venture. Why? Price-setting power arises from ownership of components which, when combined in a serial fashion, make a final composite good that consumers demand. If market power is not altered, prices will not be altered. Suppose network 2 already owns one component, BT, of a final good ASTB, and it considers the acquisition of

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[28] This is analogous to horizontally related firms underestimating the influence of their quantity changes on the opponent's quantity in the standard Cournot model.

[29] A discriminatory rate structure is not necessarily detrimental to welfare. In fact, the optimal (Ramsey) prices, that a planner would like to charge for network services, exhibit a higher markup on services that are inelastically demanded.

a second component, ST, of that same final good. This acquisition does not change the market participation of this or other firms in other final goods (such as good ASB). Network 2 has no more and no less market power after the acquisition. Its best reply to Network 1's prices p_{ASB} and q_{AS} are the *same* as before the acquisition, i.e., $q^{b}_{STB}(p_{AB}, q_{AS}) = q^{b}_{BT}(p_{AB}, q_{AS})$. This means that after the acquisition of the gateway ST, Network 2 effectively sets the price for gateway services to zero. Similarly, when network 1 owns the gateway, it also prices it at zero.

Proposition 3: As long as there are only two networks, and network 1 owns AS while network 2 owns BT, ownership of ST is of zero value to either network and will be of negative value if the fixed cost of ST is positive.

3.2 Case 2: Joint Profit Maximization, "m"

Next we consider the case in which the integrated and non-integrated networks collude on price. This situation is quite common outside of the U.S. Typically, a national firm--either privately or publicly owned--holds an end-to-end monopoly over local and long distance services, including substitute services such as cellular mobile radio.

When firms set network prices *cooperatively*, they will maximize industry profits

$$\Pi = \Pi_{ASB} + \Pi_{STB}$$

resulting in prices

$$p_{ASBm} = (ab' + a'c)/[2(bb' - c^2)],$$
$$q_{ASm} + q_{STBm} = (a'b + ac)/[2(bb' - c^2)].$$

Only the sum of prices q_{AS} and q_{STB} can be determined. Industry profits are

$$\Pi_m = (a'^2 b + a^2 b' + 2aa'c)/[4(bb' - c^2)] - F_{ASB} - F_{STB}$$

How this total is distributed between the two networks depends on their relative bargaining skills.[30] Consumer surplus and total welfare are

$$CS_m = (a'^2 b + a^2 b' + 2aa'c)/[8(bb' - c^2)],$$

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[30] A bargaining solution will lie in-between the case where Network 1 gets the total profit and the case in which it gets only its stand-alone profit under no-interconnection, while Network 2 receives the remainder. Woroch (1991) examines such Nash bargaining over interconnection terms for a simpler network setup.

$$W_m = 3(a'^2b + a^2b' + 2aa'c)/[8(bb' - c^2)] - F_{ASB} - F_{STB}.$$

3.3 Comparison of Cases 1 and 2: The Case For Collusion

We find that the price charged by network 1 does not change, i.e., $p_{ASBi} = p_{ASBm}$. This equality is a consequence of the linear cross-price effect, and does not hold in general. We also find that the price of the hybrid service ASTB is higher under competition than under joint profit maximization: $(q_{ASi} + q_{STBi}) - (q_{ASm} + q_{STBm}) = a'/(6b') > 0$. This result is caused by double marginalization in the provision of service ASTB by two independent competitors. Since price of service ASB does not change, and price of service ASTB is higher, consumers' surplus is lower under competition, $CS_i - CS_m = - 5a'^2/(72b')$. Since total profits cannot be higher than under the joint profit maximum, $(\Pi_{ASBi} + \Pi_{STBi}) - (\Pi_{ASBm} + \Pi_{STBm}) = -a'^2/(36b') < 0$, total surplus is lower under competition, $W_i - W_m = -7a'^2/(72b') < 0$. Finally, since the joint profit maximum falls short of the welfare maximum, we conclude that interconnection alone fails to achieve the optimum.

Proposition 4: The price of direct service is the same in intermodal competition and in collusion. The price of indirect service is *higher* in intermodal competition than in collusion, ensuring that consumers' surplus, profits and total welfare are *lower*.

3.4 Case 1′: Stackelberg Equilibrium, "s", and the Possibility of Foreclosure, "f"

Since network 1 is the sole provider of service AS, it could refuse access to network 2. Alternatively, it could charge a price q_{AS} so high that network 2 would find it unprofitable to operate, i.e., network 2 would be "foreclosed." To do so, network 1 must have the power to commit to such a strategy. In the last section we saw that, if the two firms moved simultaneously, then the integrated network had no desire to foreclose. In this section we consider the case where construction of the integrated network is complete. We find the appropriate price q_{AS} that causes the non-integrated network to shut down, and then calculate the best price p_{ASB} under foreclosure. Finally, we compare profits of network 1 with and without foreclosure, to see if foreclosure is part of an optimal strategy.

Before examining the profitability of foreclosure, we derive the equilibrium outcome without foreclosure. Specifically we solve for the Stackelberg equilibrium where network 1 is the leader, and both networks are in operation. We know from above that the best reply of network 2 to network 1's price pair (p_{ASB}, q_{AS}) is $q^{b}_{STB} = (a' + cp_{ASB} - b'q_{AS})/(2b')$. When this strategy is played, the profits of the two networks are

$$\Pi_{ASBs} = (2ab'p_{ASB} + a'cp_{ASB} - 2bb'p_{ASB}{}^2 + c^2p_{ASB}{}^2 + a'b'q_{AS} + 2b'cp_{ASB}q_{AS} - b'^2q_{AS}{}^2)/(2b') - F_{ASB},$$
$$\Pi_{STBs} = (a'^2 + 2a'cp_{ASB} + c^2p_{ASB}{}^2 - 2a'b'q_{AS} - 2b'cp_{ASB}q_{AS} + b'^2q_{AS}{}^2)/(4b') - F_{STB}. \qquad (*)$$

Acting as a leader, network 1 chooses p_{ASB} and q_{AS} to maximize Π_{ASBs}, assuming that firm 2 stays in operation. The Stackelberg prices for network 1 are[31]

$$p_{ASBs} = (ab' + a'c)/[2(bb' - c^2)], \quad q_{ASs} = (a'b + ac)/[2(bb' - c^2)].$$

Network 2's best reply to this price is $q_{STB} = a'/(4b')$. The profits for Networks 1 and 2 are then

$$\Pi_{ASBs} = (a'^2bb' + 2a^2b'^2 + 4aa'b'c + a'^2c^2)/[8b'(bb' - c^2)] - F_{ASB},$$
$$\Pi_{STBs} = a'^2/(16b') - F_{STB}.$$

If network 1 decides to foreclose network 2, it has to choose its prices so that the profit of network 2 in Equation (*) is zero. In fact, $\Pi_{STBs} = 0$ has two solutions, the smallest of which is equivalent to $q^{f}_{AS}(p_{ASB}) = (a' - 2\sqrt{b'F_{STB}} + cp_{ASB})/b'$. If it quotes a price just above $q^{f}_{AS}(p_{ASB})$, network 1 forecloses network 2, and its sales and profits come entirely from direct demand ASB.

Under these circumstances, what is the optimal price p_{ASB} for network 1? Once network 2 is foreclosed, $D_{ASTB} = 0$, and therefore the representative consumer has utility

$$U(D_{ASB}, D_{ASTB}) = U(D_{ASB}, 0) = \alpha D_{ASB} - \beta(D_{ASB})^2/2.$$

Maximizing $U(D_{ASB}, 0) - p_{ASB}D_{ASB}$, with respect to D_{ASB} yields the demand function for service ASB under foreclosure of ASTB[32]

$$D_{ASB} = \alpha/\beta - p_{ASB}/\beta \iff D_{ASB} = (ab' + ca')/b' - p_{ASB}(bb' - c^2)/b'.$$

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[31] p_{ASBs} and q_{ASs} are the solutions of

$$\partial\Pi_{ASBs}/\partial p_{ASB} = (2ab' + a'c - 4bb'p_{ASB} + 2c^2p_{ASB} + 2b'cq_{AS})/(2b') = 0,$$

$$\partial\Pi_{ASBs}/\partial q_{AS} = (a'b' + 2b'cp_{ASB} - 2b'^2q_{AS})/(2b') = 0.$$

[32] The inverse demand is $p_{ASB} = \alpha - \beta D_{ASB}$.

The profit function of network 1 is now $\Pi_{ASBf} = D_{ASB}p_{ASB} - F_{ASB}$. Constrained so that q_{AS} is chosen to foreclose network 2, profits of network 1 are maximized at $p_{ASBf} = (ab' + ca')/[2(bb' - c^2)]$. Note that this price is the same as without foreclosure, $p_{ASBf} = p_{ASBi}$. To implement foreclosure of the indirect service, network ASB offers to sell service AS (sold as part of ASTB) at price $q_{ASf}(p_{ASBf})$. Profits for Network 1 under foreclosure are $\Pi_{ASBf} = (ab' + ca')^2/[4b'(bb' - c^2)] - F_{ASB}$.

Collecting together the profit expressions from the various alternatives open to network 1, we conclude that foreclosure is undesirable. In fact, foreclosure yields profits less than either means for accommodating a non-integrated rival:[33]

$$\Pi_{ASBf} < \Pi_{ASBi} < \Pi_{ASBs}.$$

Notice that, as usual, network 1 would prefer to have the ability to precommit to prices before network 2 sets its price.

<u>Proposition 5</u>: Foreclosure of network 2 is a feasible but undesirable strategy of network 1.

When network 2 shuts down, network 1 experiences two effects. First, it loses sales of AS used as a component of the indirect service ASTB. Second, it gains revenues from sales of the direct service as customers substitute away from the unavailable indirect service. The integrated network must weigh the loss from losing a vertical partner against the gains of eliminating a horizontal competitor.

Recall that we found that the price of direct service was unchanged at p_{ASB} regardless of whether network 1 acted as a Stackelberg leader, a Bertrand competitor, or chose to foreclose.[34] The shift of demand toward the direct service from foreclosure will generate higher profits for direct service. But forgone sales of indirect service are greater because these

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[33] $\Pi_{ASBi} - \Pi_{ASBf} = a'^2/(9b') > 0$, and $\Pi_{ASBs} - \Pi_{ASBi} = a'^2/(72b') > 0$. Note that neither difference depends on the degree of substitutability between services.

[34] In all cases, price of direct service requires a markup inversely proportional to the elasticity of the residual demand curve. Since marginal costs are zero throughout, this says the firm operates at unit elasticity. For a linear (residual) demand curve, unit elasticity has price equal to exactly half the maximum price. Therefore, the remarkable property of all three solutions is that the price intercept of the residual demand curves for direct service is the same. The only differences across the cases is in terms of the slopes.

customers found that service to be inherently preferable at the entry-accommodating price, so they are not inclined to flock to the direct service in great numbers.

A firm that monopolizes both services would face exactly this tradeoff when considering which products to offer. Models of monopoly behavior have uncovered instances where the firm chooses not to serve some classes of consumers or some varieties of products. In an appendix, we establish that if a firm held a potential monopoly over both services, and chooses not to offer indirect service, then the integrated network would also choose to foreclose its rival. Conversely, if network ASB finds foreclosure undesirable, then a (protected) monopolist would also offer both services.[35]

4. EFFECTS OF DIVESTING THE GATEWAY AND END-LINKS

4.1 Case 3: Intermodal Competition with Competitive Interconnection, "ix"

We have seen that in intermodal competition (case 1) gateway services are sold at an effective price of zero. We now turn to the case in which an intermediate network provides the facilities that interconnect the two original networks of case 1. The gateway adds a charge of q_{ST} to the price of indirect service which is now $q_{AS} + q_{BT} + q_{ST}$.[36] The products and prices are as follows,

Service	Price	Provider
ASB (direct)	p_{ASB}	Network 1 (ASB)
AS (part of ASTB)	q_{AS}	Network 1 (ASB)
ST	q_{ST}	Network 3 (gateway ST)
BT	q_{BT}	Network 2 (BT)

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[35] The reverse of foreclosure is possible where the network 2 would shut down even though network 1 and consumers would both benefit from its existence. Here the indirect service plays the role of a "loss leader." It loses money on a stand-alone basis but its sales raises overall profits because of complementarities with the direct service. Without transfer payments, however, there is no way to induce network 2 (which holds a monopoly on STB) to offer the indirect service.

[36] Notice that, previously, q_{ST} was implicitly embedded in the total price for indirect service, i.e., $q_{AS} + q_{STB}$. Earlier, we saw that ownership of facility had no effect on competitive equilibrium or the joint profit maximum. Ownership would matter if by allocating the cost of the interconnection facility between the two networks, one or the other fail to break even.

STB	q_{STB}	Hybrid, networks 2 & 3
ASTB	$q_{AS} + q_{ST} + q_{BT}$	Hybrid, networks 1, 2 & 3

When the original two networks take pricing by the gateway as given, say at price $q_{ST} > 0$, we find that q^b_{AS} and q^b_{BT} the best reply prices of the complementary goods to ST fall with the interconnection charge, while the best reply price of network 1, p^b_{ASB}, is unaffected.[37] As expected, network profits and consumer surplus are decreasing in the interconnection charge so that network owners and users would prefer that it be set to zero.[38]

When *all three* networks choose price non-cooperatively, p_{ASB} takes the same value as before, and the component prices are

$$q_{ASix} = a'/(4b') + ac/[2(bb' - c^2)], \quad q_{BTix} = q_{STix} = a'/(4b').$$

The competitive gateway and the non-integrated network charge the same amount for their components of the indirect service because they have equal market power over the link STB.

We now compare this case with case 1, where link ST was provided by the integrated network STB. The price for the part STB of indirect service ASTB that is now divided is higher since

$$q_{BTix} + q_{STix} - q_{STBi} = a'/(6b') > 0,$$

while the price of its the complementary part AS provided by network ASB decreases,

$$q_{ASix} - q_{ASi} = -a'/(12b') < 0.$$

Summing these, we find that the price for indirect service ASTB is higher here,

$$q_{ASix} + q_{STix} + q_{BTix} - (q_{ASi} + q_{STBi}) = a'/(12b') > 0.$$

The increase in the price of STB is expected due to double marginalization from splitting STB. Although the complementary good's (AS) price decreases in response, the total effect goes in the direction of double marginalization, and thus competitive interconnection increases the price of indirect service. Since the sum of the component prices of the indirect service rises, while the

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[37] The fact that price p^b_{ASB} is unaffected is a result of the linear specification, and it may fail for general demand functions.

[38] This case would reproduce the outcome of Case 1, intermodal competition.

price of direct service remains the same, *consumers are harmed by competitive provision of gateway services.*

<u>Proposition 6</u>: Starting with intermodal competition, the introduction of competitive interconnection increases the price of indirect service, while it leaves unaffected the price of direct service. Finally, competitive interconnection causes consumers surplus to fall.

It can be shown that demand for the direct route, D_{ASB}, rises with introduction of competition for interconnection, while demand for the indirect route, D_{ASTB}, falls. The introduction of the third party causes a shift in demand from indirect to direct route. Profits for the three networks are

$$\Pi_{ASBix} = (a'^2bb' + 4a^2b'^2 + 8aa'b'c + 3a'^2c^2)/[16b'(bb' - c^2)],$$
$$\Pi_{STix} = \Pi_{BTix} = a'^2/(16b').$$

Note that, since they have the same monopoly power, Networks ST and BT charge the same price and earn the same profits.

In comparison with intermodal competition (case 1), network ASB has lower profits

$$\Pi_{ASBix} - \Pi_{ASBi} = -7a'^2/(144b') < 0.$$

This occurs despite the fact that Network ASB sets the same price for direct service as before, and it experiences higher equilibrium demand and profits from direct service since the price of indirect service has increased. However, network ASB's profits from indirect service are reduced under competitive interconnection because it has to sell AS when used as part of indirect service ASTB at a lower price than before. The reduction in profits of ASB from indirect service outstrips the increase of profits from direct service.

We know from the analysis of case 1 that, if there are only two networks, the ownership of gateway ST adds no revenue to the network that owns it, while it may add to its cost. Therefore, provided there are only two networks and there is a positive fixed cost F_{ST} associated with ST, ownership of ST is a *liability*, and each network would be willing to pay its opponent up to F_{ST} to get rid of ST. However, a third network may be willing to pay a positive amount for the gateway, up to Π_{STix}. If the original owner of ST, say network 1, divests of it by selling to a third party, its profits from operating ASB are reduced because of double marginalization from Π_{ASBi} to Π_{ASBix}. However, if there is free entry of potential entrants, network 1 can sell the gateway at a price that extracts all profits from the entrant. Then net profit to network 1 from the sale of ST is positive,

$$\Pi_{ASBix} + \Pi_{STix} - \Pi_{ASBi} = a'^2/(72b') > 0.$$

Thus, we observe <u>divestiture of interconnecting links that is profitable yet socially detrimental</u>.

Now suppose that network 2 owns ST and considers selling it off. If it is able to sell it for the maximum value it has to a third party, then it will do so because[39]

$$\Pi_{BTix} + \Pi_{STix} - \Pi_{STBi} = a'^2/(72b') > 0.$$

<u>Proposition 7</u>: Starting from intermodal competition, divestiture (sale to third party) of gateway ST is profitable to the divesting firm; yet it reduces profits for the other pre-existing network and total industry profits, and it is socially detrimental.[40]

In the case that network 1 is the original owner of ST, anticipating that network 1 would sell link ST to an third network, network 2 may try to minimize its losses by buying link ST itself. As it has already been discussed, a transfer of ST between networks 1 and 2 does not affect prices. Thus, the maximum bid that Network 2 would be willing to offer for ST equals the difference in its profits with and without competitive interconnection, i.e., $\Pi_{BTi} - \Pi_{BTix} = 7a'^2/(144b')$. This bid is smaller[41] than the bid a potential entrant (third network) which stands at $\Pi_{STix} = a'^2/(16b')$.

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[39] Note that these profitable divestitures go against the essential logic of double marginalization as presented by Cournot (1838). In his framework, the combined firm (a monopolist) was making a higher profit than the two independent vertically-related firms. Here the combined firm has lower profits than its independent parts.

[40] We can also show that total industry profits are lower with competitive interconnection. Summing the equations above we have,

$$\Pi_{ASBix} + \Pi_{STix} + \Pi_{BTix} - (\Pi_{ASBi} + \Pi_{STBi}) = -7a'^2/(144b') + a'^2(72b') = -5a'^2/(144b') < 0.$$

From Proposition 6, consumers surplus falls as a result of divestiture of ST, and so total welfare falls as well.

[41] Since $7/144 - 1/16 = -1/72 < 0$.

4.2 Case 4: Independent Ownership, "o"

Here network 1 retains a monopoly over AS, but has no stake in the SB route. This ownership structure results from case 1 by breaking up network 1 into networks 1 (operating AS) and network 3 (operating SB). As before, Network 2 serves route STB. Network 1 discriminates in price for service AS. Thus, network 1 quotes price q_{AS} when this service is used in conjunction with SB (network 3), and price q' when AS is used in conjunction with STB (network 2). Routes and corresponding prices are summarized in the following table.

Service	**Price**	**Provider**
AS (part of ASB)	q_{AS}	Network 1 (ASB)
SB (part of ASB)	q_{SB}	Network 3 (SB)
ASB	$q_{AS} + q_{SB}$	Hybrid networks 1 & 3
AS (part of ASTB)	q'_{AS}	Network 1 (ASB)
STB	q_{STB}	Network 2 (STB)
ASTB	$q'_{AS} + q_{STB}$	Hybrid, networks 1 & 2

Thus, the demand and profit functions are

$$D_{ASB} = a - b(q_{AS} + q_{SB}) + c(q'_{AS} + q_{STB}), \quad D_{ASTB} = a' - b'(q'_{AS} + q_{STB}) + c(q_{AS} + q_{SB}),$$
$$\Pi_{ASo} = q_{AS}D_{ASB} + q'_{AS}D_{ASTB} - F_{AS}, \quad \Pi_{SBo} = q_{SB}D_{ASB} - F_{SB}, \quad \Pi_{STBo} = q_{STB}D_{ASTB} - F_{STB}.$$

The non-cooperative equilibrium prices are

$$q_{ASo} = b'(3abb' + 4a'bc + ac^2)/[(9bb' - c^2)(bb' - c^2)],$$
$$q'_{ASo} = b(3a'bb' + 4ab'c + a'c^2)/[(9bb' - c^2)(bb' - c^2)],$$
$$q_{SBo} = (3ab' + a'c)/(9bb' - c^2), \quad q_{STBo} = (3a'b + ac)/(9bb' - c^2).$$

Equilibrium profits are[42]

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[42] Consumers' and total surplus are

$$CS_o = bb'(9a'^2b^2b'+9a^2bb'^2+30aa'bb'c+7a'^2bc^2+7a^2b'^2+2aa'c^3))/[2(9bb'-c^2)^2(bb'-c^2)],$$

$$W_o = (45a'^2b^3b'^2 + 45a^2b^2b'^3 + 114aa'b^2b'^2c + 5a'^2b^2b'c^2 + 5a^2bb'^2c^2 - 18aa'bb'c^3$$

$$- 2a'^2bc^4 - 2a^2b'c^4)/[2(bb' - c^2)(9bb' - c^2)^2].$$

$\Pi_{ASo} = (bb'(9a'^2b^2b' + 9a^2bb'^2 + 30aa'bb'c + 7a'^2bc^2 + 7a^2b'c^2 + 2aa'c^3)/[(9bb'-c^2)^2(bb'-c^2)] - F_{AS}$,
$\Pi_{STBo} = b'(3a'b + ac)^2/(9bb' - c^2)^2 - F_{STB}$, $\Pi_{SBo} = b(3ab' + a'c)^2/(9bb' - c^2)^2 - F_{SB}$.

One reason that network 1 (ASB) used to set a high price for AS in intermodal competition when it was used in indirect service was that it helped keep the price of ASTB high and thereby keep high the price of direct service ASB, the other product of network ASB. Once SB is divested, network 1 (now AS) has no such incentive to keep the price of AS high. Thus, in independent ownership, the price of AS used in indirect service is lower than in intermodal competition,

$$q'_{ASo} - q_{ASi} = -(a'c^2 + 3ab')/[6b'(9bb' - c^2)] < 0.$$

In response, the price of STB is higher under ownership than under intermodal competition

$$q_{STBo} - q_{STBi} = c(3ab' + a'c)/[3b'(9bb' - c^2)] > 0,$$

This is a remarkable result because it says that the price of the component that was subjected to competition by an independent firm *increases*.

Prices for composite goods are higher after divestiture of SB

$$q_{ASo} + q_{SBo} - p_{ASBi} = (3ab' + a'c)/[2(9bb' - c^2)] > 0,$$
$$q'_{ASo} + q_{STBo} - (q_{ASi} + q_{STBi}) = c(3ab' + a'c)/[6b'(9bb' - c^2)] > 0.$$

It can be checked that, at the independent ownership equilibrium, the quantities demanded of both services ASB and ASTB are *smaller* than in intermodal competition.[43] As a consequence,

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[43] Since the demand functions have a constant slope it is sufficient to check than in independent ownership equilibrium demands for both goods are smaller, i.e., that

$$-b(q_{ASo} + q_{SBo} - p_{ASBi}) + c[q'_{ASo} + q_{STBo} - (q_{ASi} + q_{STBi})] < 0,$$

and

$$-b'[q'_{ASo} + q_{STBo} - (q_{ASi} + q_{STBi})] + c(q_{ASo} + q_{SBo} - p_{ASBi}) < 0.$$

Thus, it is sufficient that

$$c/b' < [q'_{ASo} + q_{STBo} - (q_{ASi} + q_{STBi})]/(q_{ASo} + q_{SBo} - p_{ASBi}) < b/c.$$

From actual equilibrium prices we have

$$[q'_{ASo} + q_{STBo} - (q_{ASi} + q_{STBi})]/(q_{ASo} + q_{SBo} - p_{ASBi}) = 3c/b',$$

consumers surplus is lower in intermodal competition, since the demand functions have constant slope.[44]

Total profits along the direct route under independent ownership are lower than with intermodal competition,

$$\Pi_{ASo} + \Pi_{SBo} - \Pi_{ASBi} = -(3ab' + a'c)(3ab' + 5a'c)/[36b'(9bb' - c^2)] < 0,$$

as a consequence of the detrimental effects of vertical disintegration. First, after divestiture, double marginalization drives the price of route ASB to a higher level, thus reducing profits along the route. Second, Network 1 loses some of its power to price discriminate along the link AS with its horizontal interests in route SB. This reduces profits on route ASB. Since profits on route ASB are reduced as a result of divestiture, Network ASB would never voluntarily sell off SB, even if it were to receive the full rent earned by the new owner of SB.

As a response to the weaker position of the participants of route ASB after divestiture, profits of network STB are higher in independent ownership than in intermodal competition

$$\Pi_{STBo} - \Pi_{STBi} = c(3ab' + a'c)(a'(18bb' - c^2) + 3ab'c^2)/[9b'(9bb' - c^2)^2] > 0.$$

Industry profits, in contrast, may rise or fall following divestiture because

$$\Pi_{ASo} + \Pi_{SBo} + \Pi_{STBo} - (\Pi_{ASBi} + \Pi_{STBi}) =$$
$$(3ab' + a'c)(-27abb'^2 + 27a'bb'c + 15ab'c^2 + a'c^3)/[36b'(9bb' - c^2)^2].$$

Profits rise when the direct and indirect services are close substitutes (i.e., when c is relatively large), and fall otherwise. Total welfare, on the other hand, unambiguously falls,

$$W_o - W_i = (3ab' + a'c)(-189abb'^2 - 27a'bb'c + 33ab'c^2 + 7a'c^3)/[72b'(9bb' - c^2)^2)] < 0.$$

Proposition 8: **In comparison with intermodal competition, independent ownership leads to higher prices for all end-to-end services, although the price of the partial service**

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and therefore it immediately fulfills the sufficient condition.

[44] This can also be seen directly by comparing CS_o with CS_i,

$$CS_o - CS_i = (3ab' + a'c)(-135abb'^2 - 81a'bb'c + 3ab'c^2 + 5a'c^3)/[72b'(9bb' - c^2)^2)] < 0.$$

that competes with newly independent service is *higher*. Industry profits on the route whose ownership was divided are lower. Thus, network ASB never voluntarily divests SB. Profits along the indirect route are higher under independent ownership. Industry profits are higher (lower) when the direct and indirect services are close (poor) substitutes. Total welfare falls as result of divestiture of SB.

Note that if network AS owned the gateway ST (instead of it being the property of STB), the resulting prices would be identical. This observation is reminiscent of a similar comparison in Case 1. Ownership of ST by AS does not increase or decrease its monopoly power toward BT or SB. Thus, if AS owns ST, prices of all goods would be identical. It follows that the net value of link ST to Network AS is $-F_{ST}$. Reversing the argument, if ST was originally owned by Network AS, its acquisition value to Network BT would be $-F_{ST}$. However, we will see in the next section that link ST is of positive value to a third party.

4.3 Case 5: Independent Ownership with Competitive Interconnection, "ox"

As before, we now want to measure the effects of allowing a third party to supply the gateway services. Starting with the independent ownership structure of case 4, consider the divestiture of the gateway ST by network STB. After divestiture, there are four networks, each providing a single component: AS, SB, ST and BT. The charge for direct service will amount to the sum of two component prices, $q_{AS} + q_{SB}$ while the charge for indirect service includes an explicit interconnection charge, $q'_{AS} + q_{SB} + q_{ST}$.

Service	**Price**	**Provider**
AS (part of ASB)	q_{AS}	Network 1 (AS)
SB	q_{SB}	Network 3 (SB)
ASB	$q_{AS} + q_{SB}$	Hybrid networks 1 & 3
AS (part of ASTB)	q'_{AS}	Network 1 (AS)
ST	q_{ST}	Network 4 (gateway ST)
BT	q_{BT}	Network 2 (BT)
STB	q_{STB}	Hybrid, networks 2 & 4
ASTB	$q'_{AS} + q_{STB}$	Hybrid, networks 1, 4 & 2

Their profits are[45]

$$\Pi_{AS} = q_{AS}D_{ASB} + q'_{AS}D_{ASTB},\ \Pi_{SB} = q_{SB}D_{ASB},$$
$$\Pi_{BT} = q_{SB}D_{ASTB},\ \Pi_{ST} = q_{ST}D_{ASTB}$$

Equilibrium prices are[46]

$$q_{ASox} = b'(4abb' + 5a'bc + ac^2)/[2(6bb' - c^2)(bb' - c^2)],$$
$$q'_{ASox} = b(3a'bb' + 5ab'c + 2a'c^2)/[2(6bb' - c^2)(bb' - c^2)],$$
$$q_{SBox} = (2ab' + a'c)/(6bb' - c^2), \quad q_{BTox} = q_{STox} = (3a'b + ac)/[2(6bb' + c^2)],$$

We now compare this case with case 4, which was identical except for the independent provision of the ST link here.

$$q_{ASo} - q_{ASox} = b'c(3a'b + ac)/[2(9bb' - c^2)(6bb' - c^2)] > 0,$$
$$q'_{ASo} - q'_{ASox} = 3bb'(3a'b + ac)/[2(9bb' - c^2)(6bb' - c^2)] > 0,$$
$$q_{SBo} - q_{SBox} = -b'c(3a'b + ac)/[(9bb' - c^2)(6bb' - c^2)] < 0,$$
$$q_{STBo} - (q_{STox} + q_{BTox}) = - 3bb'(3a'b + ac)/[(9bb' - c^2)(6bb' - c^2)] < 0$$

As before, prices of the monopolized link AS *fall* with the introduction of competitive interconnection. Now, however, the price of SB and its substitute STB *both increase*. The price of both the direct and indirect services increase as we move to competitive interconnection,

$$q_{ASo} + q_{SBo} - (q_{ASox} + q_{SBox}) = -b'c(3a'b + ac)/2(6bb' - c^2)(9bb' - c^2) < 0,$$

$$q'_{ASo} + q_{STBo} - (q'_{ASox} + q_{STox} + q_{BTox}) = - 3bb'(3a'b + ac)/2(6bb' - c^2)(9bb' - c^2) < 0.$$

<u>Proposition 9</u>: Starting with independent ownership, divestiture of gateway ST by network STB results in lower prices of the monopolized component AS, and higher prices for SB and its substitute STB. The prices of direct and indirect service increase as a result of the divestiture.

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[45] Notice that we allow Network 1 (AS) to price discriminate between traffic coming from SB and STB.

[46] Industry profits are $\Pi_{ox} = (27a'^2b^3b'^2 + 32a^2b^2b'^3 + 74aa'b^2b'^2c + 2a'^2b^2b'c^2 - 5a^2bb'^2c^2 - 24aa'bb'c^3 - 4a'^2bc^4 - 2a^2b'c^4)/[4(bb' - c^2)(6bb' - c^2)^2]$. Consumer surplus and total welfare are $CS_{ox} = [bb'(9a'^2b^2b' + 16a^2bb'^2 + 46aa'bb'c + 16a'^2bc^2 + 9a^2b'c^2 + 4aa'c^3)]/[8(6bb' - c^2)^2(bb' - c^2)]$ and $W_{ox} = (63a'^2b^3b'^2 + 80a^2b^2b'^3 + 194aa'b^2b'^2c + 20a'^2b^2b'c^2 - a^2bb'^2c^2 - 44aa'bb'c^3 - 8a'^2bc^4 - 4a^2b'c^4)/[8(bb' - c^2)(6bb' - c^2)^2]$.

The independent provision of the gateway again reduces industry profits and harms consumer welfare.[47] Interestingly, it actually causes an increase in Network BT's component price and its profits. Apparently, competitive interconnection shifts demand to the direct service by raising the price of indirect service. Consumers and total surplus unambiguously falls, however.[48]

Proposition 10: Starting from independent ownership, the provision of gateway services by an independent firm results in smaller industry profits, consumer surplus and total welfare.

Starting with independent ownership (case 4), the incentive of network AS (if it is the original owner of gateway ST) to sell it to a third party is $\Pi_{ASox} + \Pi_{STox} - \Pi_{ASo}$. This is positive when the direct and indirect demands are of equal size (and elasticity): it becomes negative when

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[47] The difference in total profits between intermodal competition and intermodal competition with competitive interconnection is

$$\Pi_{ox} - \Pi_{o} = b'(3a'b + ac)(-135\, a'b^3b'^2 + 27ab^2b'^2c + 93a'b^2b'c^2 + 11abb'c^3$$

$$- 10a'bc^4 - 2ac^5)/[4(6bb' - c^2)^2\, (9bb' - c^2)^2] < 0.$$

The consumers surplus difference is

$$CS_{ox} - CS_{o} = (bb'(3a'b + ac)(-189a'b^2b'^2 - 135abb'^2c - 3a'bb'c^2 + 19ab'c^3$$

$$+ 4a'c^4)/[8(6bb' - c^2)^2(9bb' - c^2)^2] < 0.$$

This is seen to be negative by pairing up second and fourth terms, and third and fifth.

[48] The difference in total surplus between intermodal competition and intermodal competition with competitive interconnection is

$$W_{ox} - W_{o} = b'(3a'b + ac)(-459a'b^3b'^2 - 81ab^2b'^2c + 183a'b^2b'c^2 + 41abb'c^3$$

$$- 16a'bc^4 - 4ac^5)/[8(6bb' - c^2)^2(9bb' - c^2)^2] < 0.$$

Again, pairing the 1st and 3rd, and the 2nd and 4th terms shows this is negative. Therefore competitive supply of gateway harms total welfare. This result also follows from the comparison of profits and consumers surplus.

the indirect demand is very small and inelastic.[49] This is because of the high profits due to price discrimination that network ASB accrues before selling ST when the demand for the indirect route is very inelastic. However, starting with independent ownership, if BT was the original owner of ST, it always has a positive incentive to sell it to a third party,

$$\Pi_{BTox} + \Pi_{STox} - \Pi_{STBo} = b'(3a'b + ac)^2(9b^2b'^2 + 6bb'c^2 - c^4)/[2(6bb' - c^2)^2(9bb' - c^2)^2] > 0.$$

<u>Proposition 11</u>: If network STB is the original owner of the gateway ST, then it is always better off in selling it to a third party at the maximum asset value to this third party. If network ASB is the original owner of ST, it is better off selling it to a third party only if the demand of the indirect route is relatively elastic.

4.4 Comparison of Cases 3 and 5: Effect of Divestiture of an End-Link in the Presence of Competitive Gateways

Throughout the previous section, we assumed that gateway (ST) services were competitively supplied. We want to check the effect of breaking up network ASB into two networks, AS and SB. First of all, we find that divestiture will cause the total price for service ASB to rise because

$$(q_{ASox} + q_{SBox}) - p_{ASBix} = (2ab' + a'c)/[2(6bb' - c^2)] > 0$$

We also find that

$$q'_{ASox} - q'_{ASix} = -(2ab'c + a'c^2)/[4b'(6bb' - c^2)] < 0,$$
$$q_{BTox} - q_{BTix} = (2ab'c + a'c^2)/[4b'(6bb' - c^2)] > 0.$$

Divestiture lowers price on the AS link and raises price on the SB link. Interestingly, we find that the price charged by the gateway also rises, or

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[49]
$$\Pi_{ASox} + \Pi_{STox} - \Pi_{ASo} = b'(3a'b + ac)(54a'b^3b'^2 - 54ab^2b'^2c$$
$$- 57a'b^2b'c^2 + abb'c^3 + 7a'bc^4 + ac^5)/[4(6bb' - c^2)^2(9bb' - c^2)^2)].$$

Making the substitution $a'/a = b'/b = z$, the numerator of this expression becomes

$$a^2bz(c + 3bz)(c^5 + b^2c^3z + 7bc^4z - 54b^4cz^2 - 57b^3c^2z^2 + 54b^5z^3).$$

This is positive for $z = 1$ and negative for small z.

$$q_{STox} - q_{STix} = (2ab'c + a'c^2)/[4b'(6bb' - c^2)] > 0.$$

We now examine the price for the indirect service ASTB with and without divestiture of SB. The difference in equilibrium prices is

$$q_{ASox} + q_{STox} + q_{BTox} - (q_{ASix} + q_{STix} + q_{BTix}) = -(6a'b^2b'^2 - 8abb'^3 + 8abb'^2c - 10a'bb'^2c$$
$$+ 3a'bb'c^2 - 2ab'^2c^2 + 2ab'c^3 + a'c^4)/[4b'(bb' - c^2)(6bb' - c^2)].$$

Again using our benchmark case where $a'/a = b'/b = z$, the numerator of the price difference is

$$az(2bc^3 + c^4 + 8b^3cz + b^2c^2z - 2b^4z^2 - 10b^3cz^2).$$

This is negative for small z, and positive for large z, including $z = 1$, the case of equal demands.

Proposition 12: Under a regime of competitive interconnection, divestiture of link SB by network ASB results in higher prices for routes ASB, BT, and ST, but a lower price for AS. The overall price of service ASTB rises (falls) for relatively elastic (inelastic) demand on the indirect route.

The comparisons of profits are summarized below[50]

$$\Pi_{ASox} + \Pi_{SBox} < \Pi_{ASBix}, \;\; \Pi_{BTox} > \Pi_{STBix}, \;\; \Pi_{STox} > \Pi_{STix}.$$

Therefore, the combined profit on route ASB falls in response to divestiture of part of this route. At the same time, profits derived from both ST and BT (which are part of the alternate route ASTB) rise. Apparently, divestiture harms the competitive position of the direct service relative to the indirect service.

Finally, we can show that $CS_{ox} < CS_{ix}$ and $W_{ox} < W_{ix}$, so that consumer surplus and total welfare fall as a result of the introduction of double marginalization.[51] Industry profits compare as follows,

$$\Pi_{ox} - \Pi_{ix} = (2ab' + a'c)(-8abb'^2 + 8a'bb'c + 6ab'c^2 + a'c^3)/[16b'(6bb' - c^2)^2].$$

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[50] $\Pi_{ASox} + \Pi_{SBox} - \Pi_{ASBix} = (2ab' + a'c)(-8abb'^2 - 16a'bb'c + 2ab'c^2 + 3a'c^3)/[16b'(6bb' - c^2)^{2]} < 0$ and $\Pi_{BTox} - \Pi_{STBix} = \Pi_{STox} - \Pi_{STix} = c(2ab' + a'c)(12a'bb' + 2ab'c - a'c^2)/[16b'(6bb' - c^2)^2] > 0$.

[51] $W_{ox} - W_{ix} = (2ab' + a'c)(-56abb'^2 - 16a'bb'c + 14ab'c^2 + 5a'c^3)/[32b'(6bb' - c^2)^2]$. Pairing the first and third, and the second and fourth terms shows that this is negative, and therefore divestiture harms welfare.

This profit difference is negative when the goods are not close substitutes, and positive otherwise.

Proposition 13: Starting from intermodal competition with competitive interconnection, divestiture of route SB to a new party is undesirable to network ASB. As a result of divestiture, profits for all other networks increase, and consumer surplus and total welfare fall.

5. Concluding Remarks

In this paper we analyzed simple network competition where demanded services are comprised of components. We focus on ownership structures in which at least one component is monopolized and where imperfect substitutes exist for others. This model accurately describes several aspects of communications networks as well as the structure of other network industries as well.

In this setting, we compare prices, profits and surpluses under several different ownership structures. In all cases networks willingly interconnect. Moreover, despite its monopoly over an essential component, an integrated network prefers not to foreclose its non-integrated rivals. However, we observe that an integrated network may price its essential component higher than its end-to-end service. This practice has the flavor of a vertical price squeeze.

Our different ownership structures allows us to discuss two types of divestitures of links. First the divestiture of end-link SB is seen as a change from the ownership structure of Case 1 to that of Case 4, or a change from Case 3 to Case 5. We saw this occur with the AT&T divestiture. We find that an owner of an end-to-end network does not profit from divestiture of an end-link, even if it is able to sell the end-link at its full asset value to the new owner. Second, we consider the divestiture of the gateway ST that occurs as we move from Case 1 to Case 3, or from Case 4 to Case 5. We find that a network almost always profits by divesting the gateway, though other networks are harmed in the process. These opposite results point to the importance of the *type* of the link being divested on assessing the profitability of divestiture. With rare exceptions, we find that both kinds of divestiture reduce consumers surplus and total welfare.

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APPENDIX

Undesirability of foreclosure with general demand

Let $p_{ASTB} = q_{AS} + q_{STB}$. Without foreclosure, a representative consumer will maximize

$$U(D_{ASB}, D_{ASTB}) - p_{ASB}D_{ASB} - p_{ASTB}D_{ASTB} \tag{A1}$$

with respect to D_{ASB} and D_{ASTB} giving a system of inverse demands

$$p_{ASB} = g_1(D_{ASB}, D_{ASTB}), \quad q_{AS} + q_{STB} = p_{ASTB} = g_2(D_{ASB},D_{ASTB}). \tag{A2}$$

Inverting this system produces the corresponding demand functions,

$$D_{ASB}(p_{ASB}, q_{AS} + q_{STB}), \quad D_{ASTB}(q_{AS} + q_{STB}, p_{ASB}). \tag{A2'}$$

Thus, *without* foreclosure, the profit functions for the two networks are:

$$\Pi_{ASB}(p_{ASB}, q_{AS}, q_{STB}) = \Pi_{ASBi}(p_{ASB}, q_{AS} + q_{STB}) = p_{ASB}D_{ASB} + q_{AS}D_{ASTB} - F_{ASB}, \tag{A3a}$$

$$\Pi_{STB}(p_{ASB}, q_{AS}, q_{STB}) = \Pi_{STBi}(p_{ASB}, q_{AS} + q_{STB}) = q_{STB}D_{ASTB} - F_{STB}. \tag{A3b}$$

Turning to the case of foreclosure, network ASB quotes a high enough price q_{AS} to make profits of network ASTB zero, so that ASTB withdraws from the market.[52] When ASTB withdraws, sales of the indirect service are zero, i.e., $D_{ASTB} = 0$. Maximizing (A1) with respect to D_{ASB}, holding $D_{ASTB} = 0$, gives the inverse demand for ASB under foreclosure:

$$p_{ASB} = g_1(D_{ASB}, 0). \tag{A4}$$

Inverting (A4) gives the demand function for network ASB under foreclosure, $D^f_{ASB}(p_{ASB})$. This demand is exactly the demand for network 1 in the case when both networks are in service (inverse demand system (A2)) but the price of service ASTB is so high that demand for its service is zero. For given p_{ASB}, define $p^z_{ASTB}(p_{ASB})$ to be the price of ASTB that makes demand D_{ASTB} zero, i.e.,

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[52] If $F_{STB} = 0$, the price of AS when used as part of ASTB to accomplish foreclosure is $q_{AS} = p^z_{ASTB}(p_{ASB}) - q_{STB}$, since foreclosure happens at zero demand. If there is a positive fixed cost of network 2, $F_{STB} > 0$, the required q_{AS} to accomplish foreclosure is smaller, $q_{AS} < p^z_{ASTB}(p_{ASB}) - q_{STB}$, since foreclosure is accomplished at positive demand for ASTB.

$$D_{ASTB}(p^{z}_{ASTB}(p_{ASB}), p_{ASB}) = 0.$$

Then we can write the condition defining foreclosure demand as

$$D^{f}_{ASB}(p_{ASB}) = D_{ASB}(p_{ASB}, p^{z}_{ASTB}(p_{ASB})).$$

Then profits of network 1 under foreclosure are

$$\Pi^{f}_{ASB}(p_{ASB}) = p_{ASB}D^{f}_{ASB}(p_{ASB}) - F_{ASB} = p_{ASB}D_{ASB}(p_{ASB}, p^{z}_{ASTB}(p_{ASB})) - F_{ASB} =$$
$$= \Pi_{ASBi}(p_{ASB}, p^{z}_{ASTB}(p_{ASB})) = \Pi_{ASB}(p_{ASB}, p^{z}_{ASTB}(p_{ASB}) - q_{STB}, q_{STB}),$$

where $q_{AS} = p^{z}_{ASTB}(p_{ASB}) - q_{STB}$. Now

$$\max_{p_{ASB}} \Pi^{f}_{ASB}(p_{ASB}) \leq \max_{p_{ASB},q_{AS}} \Pi_{ASB}(p_{ASB}, q_{AS}, q_{STB}), \qquad (A5)$$

since q_{AS} is unconstrained on the RHS. Therefore foreclosure is never strictly preferred to competition. Foreclosure can only happen when equation (A5) holds with equality, i.e., when the unconstrained optimal price for q_{AS} makes the demand for ASTB zero. Even then, network ASB is just indifferent between foreclosing and not foreclosing.

The profits of a monopolist providing both services be $\Pi(p_{ASB}, q_{AS}) = \Pi_{ASB}(p_{ASB}, q_{AS}, 0) + \Pi_{STB}(p_{ASB}, q_{AS}, 0)$. A monopolist providing both services will realize at least the profits of ASB when both products are offered. However, under foreclosure, the monopolist's profit's are the same as that of ASB. Therefore we can extend (A5) to

$$\max_{p_{ASB}} \Pi^{f}_{ASB}(p_{ASB}) \leq \max_{p_{ASB},q_{AS}} \Pi_{ASB}(p_{ASB}, q_{AS}, q_{STB}) \leq \max_{p_{ASB},q_{AS}} \Pi(p_{ASB}, q_{AS}) \qquad (A6)$$

When a monopolist forecloses, the RHS is equal to the LHS, and therefore necessarily the expression in the middle is equal to both. It follows that network ASB also chooses to foreclose.