The Incentive of a Multiproduct Monopolist to Provide All Goods

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Abstract

This note shows that a monopolist facing any **linear** demand system for n goods and no fixed costs will produce positive quantities of all goods as long as demand is positive for all goods when all are sold at marginal cost. This is in contrast with the traditional view that, in general, a multiproduct monopolist does not produce positive quantities of all goods even though there is positive demand for each of them when prices are equal to marginal cost.

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It is well accepted in the literature that, in general, a multi-product monopolist may choose not to produce positive quantities of all goods. This arises because pricing of one good "interferes" with pricing of other goods. In particular, if a good of low value is offered at a low price while a good of high value is offered at a high price, consumers that would have bought the high value good if both goods were offered at marginal cost may now buy the low value good. This imposes constraints on the monopolist who may find it better not to offer the low value good at all.[1]

In contrast, this note shows that a monopolist facing any linear demand system (derived from a quadratic utility function) will produce positive quantities of all goods. This is true despite potentially very significant differences in the willingness to pay for different goods. The only requirement for this result is that positive quantities are demanded for all goods when they are all sold at marginal cost. Thus, essentially this paper shows that the "interference" between high and low value goods in the pricing of the monopolist never happens for linear demand systems.

Let a single consumer have a quadratic utility function in $x_1, x_2, ..., x_n$, that is separable in the outside good x_0, i.e,

$$U(x_0, x_1, ..., x_n) = x_0 + \Sigma^n_{i=1} \alpha_i x_i - [\Sigma^n_{i=1} \beta_i x_i^2 + 2\Sigma_{i\neq j} \gamma_{ij} x_i x_j]/2.$$

[1] See Mussa and Rosen (1978).

with $\alpha_i, \beta_i, \gamma_{ij} > 0$. Maximization of $U(x_0, x_1, ..., x_n)$ subject to $x_0 + \Sigma_{i=1}^n p_i x_i = I$ yields linear inverse demand functions[2,3]

$$p_i = \alpha_i - \beta_i x_i - \Sigma_{i\neq j} \gamma_{ij} x_j, \quad i = 1, ..., n. \tag{1}$$

For convenience define the price, sales, intercept, and marginal cost vectors as $\mathbf{p} = (p_1, ..., p_n)$, $\mathbf{x} = (x_1, ..., x_n)$, $\alpha = (\alpha_1, ..., \alpha_n)$, and $\mathbf{c} = (c_1, ..., c_n)$.

We assume that the system of inverse demand functions (1) is invertible; i.e., that the solution of

$$\beta_i x_i + \Sigma_{i\neq j} \gamma_{ij} x_j = k_i, \quad i = 1, ..., n, \tag{2}$$

where $k_i = \alpha_i - p_i$, exists and is unique. Let the general solution of the system be expressed as $x_i = f_i(\mathbf{k})$, i = 1, ..., n;[4] in particular demand for good i is $f_i(\alpha - \mathbf{p})$. As a solution to a system of linear equations, function $f_i(\mathbf{k})$ is linear in $\mathbf{k}$.

Suppose that a monopolist can produce all these goods without fixed costs, and the marginal cost for good i is c_i. The profits of the monopolist are

$$\Pi = \Sigma_{i=1}^n x_i(p_i(\mathbf{x}) - c_i).$$

The marginal profit in x_i is

$$\partial\Pi/\partial x_i = p_i - c_i + x_i(\partial p_i/\partial x_i) + \Sigma_{i\neq j} x_j(\partial p_j/\partial x_i)$$

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[2] Standard second order conditions apply.

[3] The derivation of the demand system from utility maximization of a single consumer ensures that the cross partials of the demand functions are equal, $\partial p_i/\partial x_j = \partial p_j/\partial x_i = -\gamma_{ij}$.

[4] Using our convention, $\mathbf{k} = (k_1, ..., k_n)$.

$$= p_i - c_i - \beta_i x_i - \Sigma_{i \neq j} \gamma_{ij} x_j = 2p_i - \alpha_i - c_i. \tag{3}$$

by substitution from (1) in the last two equalities.[5] Therefore, at the candidate profit maximum, defined by the first order conditions $\partial\Pi/\partial x_i = 0$, $i = 1, ..., n$, prices are

$$p_i^* = (\alpha_i + c_i)/2, \; i = 1, ..., n. \tag{4}$$

Substituting the candidate equilibrium prices in the demand system (2), we find the candidate equilibrium sales in each good,

$$x_i^* = f_i(\alpha - \mathbf{p}^*) = f_i((\alpha - \mathbf{c})/2).$$

Are these sales positive for every good? To see if this is true, observe that, since function f_i is linear, scalars commute, and therefore

$$x_i^* = f_i((\alpha - \mathbf{c})/2) = f_i(\alpha - \mathbf{c})/2. \tag{5}$$

Since, in general, given prices $\mathbf{p}$, the demand for good i is $f_i(\alpha - \mathbf{p})$, equation (5) says that the candidate equilibrium sales for each good are exactly half of the demand when every good is offered at marginal cost, i.e., when $\mathbf{p} = \mathbf{c}$. Therefore, if every good has positive sales when all goods are sold at marginal cost, then the quantity of each good sold at the candidate equilibrium is positive.

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[5] We also used the fact that in the quadratic utility function $\gamma_{ji} = \partial p_j/\partial x_i = \partial p_i/\partial x_j = \gamma_{ij}$.

<u>Theorem</u>: If all goods have positive demands when sold at marginal cost and the demand system is linear, then a monopolist provides positive quantities of all goods at equilibrium.

We need to stress here that the theorem is correct for *any* linear demand system, even where the willingness to pay for some good is much higher than the willingness to pay for some other good. What happened to the "interference" in pricing from low value goods to high value goods? Clearly, this interference is only present when the demand system is non-linear. Although our results do not diminish the importance of "interference" in the maximization problem of the monopolist facing non-linear demands, it shows that for the important class of linear demand systems, a monopolist will provide all goods that would have been provided under perfect competition. For such demand systems, monopoly does not result a distortion of variety; it only results in the traditional allocative distortion through lower production.

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