Desirability of Compatibility in the Absence of Network Externalities

By NICHOLAS ECONOMIDES*

I compare the incentives firms have to produce individual components compatible with components of other manufacturers instead of "systems" composed of components that are incompatible with components of competing manufacturers. I show that, even in the absence of positive consumption externalities ("network" externalities), prices and profits will be higher in the regime of compatibility. Equilibrium total surplus could be higher in either regime. Both regimes overprovide variety compared to the surplus-maximizing solution.

In today's environment, many products are complex, specified by a long array of characteristics. Part of the decision of the firm is to choose if it should break this long array of features into two or more smaller parts, and produce the products corresponding to the smaller arrays, instead of the product that embodies the full array. We will call the products that correspond to these smaller arrays *components*, and the product that corresponds to the full array a *system*. Note that components are by their definition *complements*.[1]

Components produced by different manufacturers are *compatible* if it is feasible for the consumers to combine them costlessly into a working system. In this paper we will assume that there are two components which make up a system. We will discuss competition under full compatibility (when components produced by all firms are compatible with each other) and under incompatibility (when no components produced by different firms are compatible and no hybrid systems can exist).[2] Assuming availability of the same

*Columbia University, New York, NY 10027 and Stanford University, Stanford, CA 94305. I thank Tina Fine, Andreu Mas-Colell, Roy Radner, Susan Rose-Ackerman, and two anonymous referees for their comments and suggestions. Research support from the National Science Foundation under grant no. SES-8408905 is gratefully acknowledged.

[1]It should be clear that a system cannot be broken into arbitrary components, and it may be impossible to break some systems into components. However, there are many examples of products that can be broken into components. Stereo systems can be broken into three components: receiver, amplifier, and speakers, or alternatively into two components: receiver-amplifier and speakers. A personal computer can be broken into two components, monitor and central unit, and this is in fact the way most non-portable MS-DOS systems are sold. Alternatively, personal computers can be sold as systems, as was the original *Macintosh* by *Apple*. Indeed, personal computers are typically made of a number of compatible components, including the "mother-board," disk drives, disk-drive controllers, video output card, input/output card, etc. It is an interesting fact that in the first PC line, IBM had itself manufactured only the "mother-board," and limited itself to the assembly of the other components.

[2]Carmen Matutes and Pierre Regibeau (1988) discuss a similar duopoly model. In their model, consumers are located on a square according to the peaks of their utility functions. Consumers at every position have a $0-1$ demand function of the delivered price. Transportation costs are proportional to the sum of the coordinate differences (block metric). In the regime of incompatibility, the two systems are located at the opposite corners on the diagonal of the square, while, under compatibility, systems at all four corners are available. Using direct computation, they show that prices and profits are higher in the regime of compatibility. Our paper shows that prices and profits are higher under compatibility for general consumers' demand and transportation cost (disutility of distance) functions. Our method of proof is different. We express the demand a firm faces in each regime in terms of a "standard demand in a market of width A" (see Section II) and thereby we are able to compare prices and profits across regimes without using a specific functional form for consumers' demand at each location or for their disutility of distance. We describe competition among n firms, and in Section VII we show that higher

technology to all firms, we show two very strong results. First, that, for any given number of firms n, *equilibrium prices are higher under compatibility*. Second, that, for given n, *equilibrium profits are higher under compatibility*. These results are established although we explicitly rule out positive consumption externalities (network externalities) that would naturally lead to similar conclusions.[3]

The intuition behind the pricing result is quite simple. Suppose that a firm faces the same demand function in both environments. Suppose that a firm cuts the price of its component No. 1 under compatibility by Δp and similarly shaves the price of its system by Δp under incompatibility. In the regime of compatibility the demand response is in units of component No. 1, while in the regime of incompatibility the demand response is in units of a system composed of component No. 1 *and component No. 2*. Thus, an equal price cut will lead to a higher value response under incompatibility. This signifies that the residual demand under incompatibility is more elastic. Therefore competition is more intense under incompatibility, and lower prices will prevail in that regime. This argument is refined in Section IV to take into account natural differences in the residual demand faced by a firm in the two regimes. Since the number of available systems (component combinations) is much larger under compatibility, the "market area" of any system is significantly lower in this regime. We show that this leads to even lower prices under incompatibility.

Higher profits under incompatibility for any given number of firms n implies that the free-entry equilibrium number of firms will be higher in that regime. A larger number of competitors intensifies competition, and in general may reverse the comparison of prices across the two regimes. However, we show in Section VII that for a well-established class of demand and transportation cost functions the price comparison is preserved under free entry.

The organization of the paper is as follows. Section I presents the basic model of differentiated products in two dimensions of variety. Section II establishes the equilibrium in the regime of compatibility. Section III describes the equilibrium under incompatibility. In Section IV I compare the equilibrium prices of the two regimes. In Section V the equilibrium profits of the two regimes are compared. Section VI discussed free entry. Section VII analyzes an important special case where demand is inelastic and transportation cost functions are quadratic. In this section I also characterize optimal diversity and compare it with the free-entry equilibria of the two regimes. In Section VIII I conclude.

I. The Setup

Suppose that a *system* is composed of two *components*, No. 1 and No. 2. I assume that each component is of no value to a consumer unless he also possesses a complementary component. Consumers have differentiated preferences over the features of each component, and therefore over the features of a system. Two situations are envisioned. In the first, there is full compatibility. Any component of type 1 is compatible with any component of type 2, and together they make a feasible system. This can be achieved when there are known and accepted standard specifications to which all firms adhere. In an alternative environment there is complete incompatibility. No component of type 1 can be combined with a component of type 2

prices and profits in the compatibility regime are maintained under free-entry conditions.

[3]Consumption can create a positive externality when it enhances the value of a complementary good that has some public good features. For example, the purchase of a VHS video tape player increases the value of the library of VHS films, which in turn increases the value of another VHS player unit. In that context, producing a product that is compatible with large groups of other products is desirable because of the direct enhancement to its value afforded by the network externality. In this paper we abstract away from any such positive consumption externalities. See Katz and Carl Shapiro (1985) for an extensive discussion of network externalities.

unless they are manufactured by the same firm.[4]

A differentiated good is a two-dimensional object (x_1, x_2), where x_1 is its specification in the dimension of component No. 1, and x_2 is its specification in the dimension of component No. 2. Specifications of goods lie in a two-dimensional space of characteristics. Consumers have single-peaked and diverse preferences in the space of characteristics.[5] Thus, consumers can be grouped according to the specification they like most, and can be thought of as residing at their most preferred point. Products at distance "d" from the most preferred position of a consumer are valued at $f(d)$ less than the most preferred good, where $f(d)$ is an increasing and convex function passing through the origin.

Except for the differentiated products, there exists only one other "outside" good that represents all other goods (Hicksian composite good). Formally, the total utility of consumers whose most preferred bundle is $\mathbf{z} = (z_1, z_2)$, endowed with m units of the outside good, when they buy q units of product "$\mathbf{x}$," is

$$U_z(q, \mathbf{x}, p, m) = m + V_z(q, \hat{p}),$$

where

$$V_z(q, \hat{p}) = k(q) - q\hat{p}$$

is net the utility from the consumption of the differentiated product.

$$\hat{p}(p, \mathbf{x}, \mathbf{z}) = p + f(\|\mathbf{x} - \mathbf{z}\|)$$

is the utility cost of a unit of $\mathbf{x}$ to a consumer of type $\mathbf{z}$. In the locational interpretation, $\hat{p}$ is the "delivered price" to a consumer "residing" at $\mathbf{z}$. $k(q)$ measures the total willingness to pay for q units by consumers residing at $\mathbf{z}$.[6] For expositional purposes we define the composition of the transportation cost function $f(\cdot)$ and the distance function $\|\cdot\|$ as $g(\cdot)$:

$$g(\mathbf{a}) \equiv f(\|\mathbf{a}\|).^7$$

Consumers decide which product to buy, and how many units of it. All consumers located at z buy the same product, the one that minimizes $\hat{p}$. Maximizing the utility function U_z with respect to quantity, q, implies

$$\hat{p} = k'(q),$$

and therefore the demand of consumers located at z is $q = k'^{-1}(\hat{p}) \equiv X(\hat{p})$.[8]

Consumers are distributed uniformly accordingly to their most preferred variety on a surface of a sphere that has a great circle of length 1. Consumers receive utility from the consumption of a "system," that is, from a pair of components of types No. 1 and No.

[4]An intermediate situation could result if components of type 1 made by different firms do not necessarily follow the same specifications, but the manufacturer of one of the components provides free of charge an *adapter* or *interface* which allows compatibility. This case opens the possibility for firms to compete in the pricing of the interface and to attempt to discriminate. We leave this case open for further research.

[5]Product variation in two characteristics has been discussed by B. Curtis Eaton and Richard Lipsey (1980), Economides (1986), and Frank Fetter (1924) among others.

[6]A distance function $\|\cdot\|$ from $\mathbf{R}^2$ to $\mathbf{R}_+$ maps the vector of differences in coordinates to nonnegative real numbers. It fulfills $\|\mathbf{0}\| = 0$, $\|\mathbf{a}\| = \|-\mathbf{a}\|$, $\|\mathbf{a}\| > 0$ for $\mathbf{a} \neq \mathbf{0}$, and $\|\mathbf{a}\| + \|\mathbf{b}\| \geq \|\mathbf{a} + \mathbf{b}\|$. The disutility of distance (transportation cost function) is an increasing and (weakly) convex function of distance, that is, $f'(d) > 0$, $f''(d) > 0$, and $f(0) = 0$.

[7]$g(\mathbf{a})$ is not necessarily a distance function, in the sense that it may not follow the triangle inequality. Some commonly used disutility of distance functions, such as $f(d) = kd$ and $f(d) = d/(1+d)$, result (in composition with a distance function $d(\mathbf{a})$) in transportation cost functions $g(\mathbf{a})$ that fulfill the triangle inequality. However, others, including the commonly used quadratic transportation cost function $f(d) = d^2$, can result in functions $g(\mathbf{a})$ that may not fulfill the triangle inequality. For example, $f(d) = d^2$ applied to the Euclidean distance function results in $g(\mathbf{a}) = a_1^2 + a_2^2$, which fails the triangle inequality for any three points that do not form a right-angled triangle.

[8]In the special case of inelastic demand where every consumer buys one unit (a la Harold Hotelling, 1929) the utility function is

$$U_z(x, p, m) = k + m - p - f(\|x - z\|).$$

2. We assume that the technology of production is the same for all firms, and consists of constant marginal costs for each component and setup costs F_1 and F_2 for components No. 1 and No. 2, respectively. We assume no economies of scope so that the setup cost for the production of systems is $F = F_1 + F_2$. Below we use the symbol p to denote the differential of price over and above constant marginal cost. We will call them prices, as if marginal costs were zero, but the equivalence with the case of positive constant marginal cost is obvious.

II. Equilibrium Under Compatibility

Consider first full compatibility. Let n firms produce each component. Let component No. 1 be located on the vertical axis, while component No. 2 is located on the horizontal axis. See Figure 1. We assume that single-maker systems (each composed of components made by the same firm) are located symmetrically on the diagonal at positions $\dots(-d,-d),(0,0),(d,d),\dots$. Hybrid systems are composed of components made by different firms. They lie on the nodes of a grid of width and length d. A firm derives profits from sales of component 1 (horizontal demand, HD) and from sales of component 2 (vertical demand, VD).

Next we discuss a symmetric equilibrium. Let all other firms, except firm 1, offer components No. 1 and No. 2 at prices $\bar{p}_1$ and $\bar{p}_2$, Firm 1 quotes prices p_1 and p_2, respectively, for components No. 1 and No. 2. The demand for its component No. 1, HD, comes from consumers who buy the other component from any of $n-1$ other firms as a part of a hybrid system, and from consumers who buy it as a part of the single-maker system sold by firm 1. Consumers in the first category are located in $n-1$ areas like the shaded box in Figure 1. The demand for component No. 1 in each box can be written as $D(p_1+\bar{p}_2, \bar{p}_1+\bar{p}_2; d)$, where $p_1+\bar{p}_2$ is the total price consumers pay for the hybrid system they buy, and $\bar{p}_1+\bar{p}_2$ is the total price of a competing neighboring hybrid system. The parameter d denotes the extent of the market, that is, the horizontal width of the shaded box.

In general, $D(p_j, p; A)$ denotes the demand of firm j quoting price p_j while all other firms quote p, when all firms are equispaced at distances d on a straight line, and consumers are uniformly distributed according to their most preferred products on a strip of width A across the line of location of firms. See Figure 2. We call $D(p_j, p; A)$ the "standard demand in a market of width A."

The demand for component No. 1 from consumers that buy both components from firm 1, located in Figure 1 in the square of the dotted lines around (0, 0), is $D(p_1+p_2, \bar{p}_1+p_2; d)$. Therefore total demand for component No. 1 is

$$\begin{aligned}\text{HD}(p_1, p_2, \bar{p}_1, \bar{p}_2) &= (n-1)D(p_1+\bar{p}_2, \bar{p}_1+\bar{p}_2; d)\\ &\quad + D(p_1+p_2, \bar{p}_1+p_2; d).\end{aligned}$$

Similarly, the demand for component No. 2 is

$$\begin{aligned}\text{VD}(p_1, p_2, \bar{p}_1, \bar{p}_2) &= (n-1)D(\bar{p}_1+p_2, \bar{p}_1+\bar{p}_2; d)\\ &\quad + D(p_1+p_2, p_1+\bar{p}_2; d).\end{aligned}$$

Therefore, the profit function of firm 1, in the regime of compatibility is

$$\Pi^C(p_1, p_2, \bar{p}_1, \bar{p}_2) \equiv p_1\text{HD} + p_2\text{VD} - F_1 - F_2.$$

Maximization with respect to p_1 implies

$$\begin{aligned}0 = \Pi_1^C(p_1, p_2, \bar{p}_1, \bar{p}_2) &\equiv \text{HD} + p_1(n-1)D_1(p_1+\bar{p}_2, \bar{p}_1+\bar{p}_2; d)\\ &\quad + p_1 D_1(p_1+p_2, \bar{p}_1+p_2; d)\\ &\quad + p_2[D_1(p_1+p_2, p_1+\bar{p}_2; d)\\ &\qquad + D_2(p_1+p_2, p_1+\bar{p}_2; d)],\end{aligned}$$

where subscripts of D and Π^C denote partial derivatives. At a symmetric equilibrium $p_1 = p_2 = \bar{p}_1 = \bar{p}_2 = \bar{p}(d)$, where $\bar{p}$ is expressed as a function of the width of the

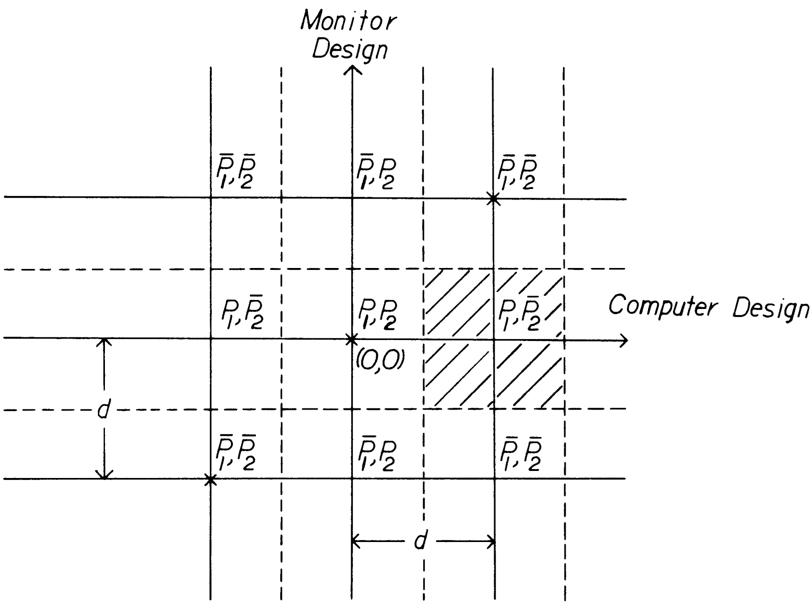

FIGURE 1. COMPATIBLE COMPONENTS

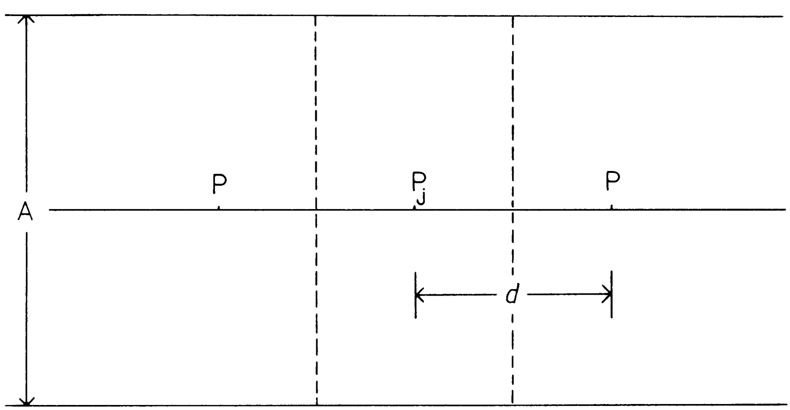

FIGURE 2. "STANDARD" DEMAND IN A STRIP OF WIDTH A. LOCATION OF SYSTEMS COMPOSED OF INCOMPATIBLE COMPONENTS FOR $A = 1$

market. Substituting in the first-order condition and dividing by n we deduce that $\bar{p}(d)$ solves,

$$(1)\quad D(2\bar{p},2\bar{p};d)+\bar{p}D_1(2\bar{p},2\bar{p};d) + \bar{p}\left[D_1(2\bar{p},2\bar{p};d) + D_2(2\bar{p},2\bar{p};d)\right]/n = 0.$$

Equilibrium price for a system under compatibility is[9]

$$p^C = 2\bar{p}(d).$$

[9]Maximizing with respect to p_2 under symmetry results in the same condition (1).

Equilibrium profits are

$$\Pi^{C*} = \Pi^C(p^C/2, p^C/2, p^C/2, p^C/2)$$
$$= p^C D(p^C, p^C; d)/d - F.$$

III. Equilibrium Under Incompatibility

Let there be n "systems" available, each comprised of a component of type 1 and a component of type 2. Any component of type 1 is incompatible with any component of type 2 which is not made by the same firm. Thus, under incompatibility the competing products are "systems" rather than components. Competing products (indicated by x) are located equidistantly on a diagonal of the grid in Figure 1. Figure 2 for $A=1$ shows the locations of the systems, located d apart, from a point of view rotated 45°. The perpendicular "width" of the area where consumers are located is 1. Thus, firms face "standard demand" in a market of width 1.

When all other firms sell a system at $\tilde{p}$, the profits of a firm quoting p are

$$\Pi^I(p, \tilde{p}) = pD(p, \tilde{p}; 1) - F.$$

Assuming concavity of Π^I in p, maximization implies

$$\Pi_1^I(p, \tilde{p}) = D(p, \tilde{p}; 1) + pD_1(p, \tilde{p}; 1) = 0.$$

At a symmetric equilibrium $p = \tilde{p}(1)$, where $\tilde{p}$ is expressed as a function of the width of the market. $\tilde{p}(1)$ solves

$$(2) \quad D(\tilde{p}, \tilde{p}; 1) + \tilde{p}D_1(\tilde{p}, \tilde{p}; 1) = 0.$$

The price of a system at the equilibrium of the incompatibility is

$$p^I = \tilde{p}(1),$$

and equilibrium profits are

$$\Pi^{I*} = \Pi^I(p^I, p^I) = p^I D(p^I, p^I; 1) - F.$$

IV. Comparison of Equilibrium Prices

Comparing equations (1) and (2) we see two differences. First, $2\bar{p}$ and $\tilde{p}$ enter the first two terms of (1) and (2) in slightly different ways. The nature of competition is crucially determined by this difference as seen below. Second, the width of the market under incompatibility is 1, while under compatibility the problem is expressed in terms of standard demand in a market of width d. This second difference is a consequence of the fact that available product combinations lie much closer to each other in the specification space under compatibility than under incompatibility. The effective extent of the market differs across the two cases.

We discuss the two differences separately. To abstract from the second one we consider a fictitious market structure under incompatibility. Let firms be located d apart in a market of width A. When all other firms charge $\tilde{p}$, a firm charging p has profit function

$$\Pi^f(p, \tilde{p}; A) = pD(p, \tilde{p}; A) - F.\text{[10]}$$

The first-order condition at a symmetric equilibrium is

$$(2') \quad D(\tilde{p}, \tilde{p}; A) + \tilde{p}D_1(\tilde{p}, \tilde{p}; A) = 0.$$

Let its solution be $\tilde{p}(A)$. We are interested in the fictitious market of width $A = d$, so that the effective extents of the markets are equal under compatibility and incompatibility.

We now show that

$$(3) \quad p^C \equiv 2\bar{p}(d) > \tilde{p}(d),$$

so that, *in markets of equal extent, firms have higher prices under compatibility*. Evaluating Π_1^C at $p_1 = p_2 = \bar{p}_1 = \bar{p}_2 = \tilde{p}(d)/2$, and substituting $D(\tilde{p}, \tilde{p}; A)$ from (2′) at $A = d$ yields

$$\Pi_1^C(\tilde{p}/2, \tilde{p}/2, \tilde{p}/2, \tilde{p}/2)$$
$$= \tilde{p}[(1-n)D_1(\tilde{p}, \tilde{p}; d) + D_2(\tilde{p}, \tilde{p}; d)]/2 > 0.$$

[10]Of course $\Pi^f(p, \tilde{p}; 1) = \Pi^I(p, \tilde{p})$.

This is positive because D_1 is the negative slope of the demand and D_2 is positive because the demand increases in the price of the substitute good. From (1), we have $\Pi_1^C(\bar{p}, \bar{p}, \bar{p}, \bar{p}) = 0$. Assuming

$$\sum_{j=1}^{4} \Pi_{1j} < 0,$$

a common assumption that guarantees uniqueness of the noncooperative equilibrium, (3) follows immediately. Therefore, when the markets are of the same effective size, prices are higher under compatibility.

The intuition for this result is simple. Starting from the same levels of prices and demand, consider a price increase in one component that produces the same decrease in demand under both compatibility and incompatibility. Under incompatibility the loss of profits is higher because *systems* sales are lost rather than sales of *one component*. Thus, profits are more responsive to price under incompatibility than under compatibility. This is another way of saying that the residual demand facing a firm is more elastic under incompatibility. As a consequence, firms are going to choose lower prices under incompatibility.

The situation is reminiscent of Augustin Cournot's (1927, pp. 99–103) price-quoting monopolists in complementary goods.[11] Cournot assumes that one unit of brass is produced by costlessly combining one unit of copper and one unit of zinc. When the demand for brass is $D(p)$, the symmetric equilibrium of two competing monopolists in zinc and copper is described by

$$\text{(C)} \qquad D(p) + pD'(p)/2 = 0.$$

However, a monopolist of *both* zinc and copper will sell at the price that solves

$$\text{(C}'\text{)} \qquad D(p) + pD'(p) = 0.^{12}$$

Equation (C) resembles equation (1) ($p = 2\bar{p}$) except for its last term, while equation (C′) resembles (2′) at $A = d$ ($p = \tilde{p}$). Cournot observes that "the root of equation (C) is always greater than that of equation (C′)...." The intuitive reason is not given by Cournot, but it is exactly the same as the one given above. A drop in the price of one of the ingredients (say zinc) increases equally the demand for zinc in the case of independent monopolists as it increases the demand for both zinc and copper in the case of a single monopolist of both metals. Therefore equal price reductions result in a larger revenue increase for the "fused monopolist." Facing a more elastic demand, the "fused monopolist" chooses lower prices than the two independent monopolists.[13]

In view of equation (3), to complete the comparison between p^C and p^I, we now compare $\tilde{p}(1)$ and $\tilde{p}(d)$. This comparison is necessary because the distances between neighboring products under compatibility are significantly lower than under incompatibility. We find conditions such that $\tilde{p}(A)$ is decreasing in A. Then $p^I \equiv \tilde{p}(1) < \tilde{p}(d)$ since $d < 1$, and combining with (3) we will have $p^I < p^C$. $\tilde{p}(A)$ decreasing in A means that as we increase the width of the market perpendicular to the axis of location of the firms the resulting equilibrium price decreases. This means that competition for customers located further away from the firms is more intense than for consumers located closer to the axis.[14]

[11] In fact this problem is the dual of the standard quantity-setting Cournot duopoly in substitute goods. See Hugo Sonnenschein (1968) and Theodore Bergstrom (1988).

[12] This is immediate from the fact that the monopolist of both markets maximizes $\Pi^b = (p_z + p_c) D(p_z + p_c)$ while the independent monopolist of zinc maximizes $\Pi^z = p_z D(p_z + p_c)$ and assumes that $dp_c/dp_z = 0$.

[13] Equations (1) and (C) differ in the term in brackets of (1) that does not appear in (C). This term comes from the fact that under compatibility a firm sells both components to some customers. In Cournot, in the case of competing monopolists, each monopolist had exclusive production of his metal. The fact that we are able to show that, in markets of the same extent, prices will be higher under compatibility suggests that the conclusion of Cournot for homogeneous goods can be extended to the comparison of the "fused" monopolist (who produces all of both metals) to duopolists producing some (but not all) of each metal.

[14] More precisely, competition is more intense for consumers located away from the axis of location of the firms.

As in every model of locational differentiation, here too when a firm increases its price it loses customers on two margins. In the first margin customers are lost to neighboring firms as the boundary of the market area of the firm moves closer to its location. In the second margin, customers are lost to the "outside" good as they decide to switch from consumption of a differentiated product to consumption of the homogeneous good. It is useful to analyze the effects of the increase in the width of the market strip separately for each margin.[15] In Part A we analyze the effects of the first margin which are determined by the locational variation of the boundary dividing the market areas of the firms. In Part B we analyze the effect of the second margin which depends on the elasticity of demand generated by consumers "located" at any point z.

A. *Inelastic Demand*

We start the analysis of the first margin by assuming that the demand is inelastic so that the second margin is zero. Let *all* consumers buy one unit of a differentiated product.[16] Consider the boundary dividing the market areas of two neighboring firms. Normalizing the firms' locations at $(a,0)$ for firm 1 and at $(-a,0)$ for firm 2, this boundary is the locus of $\mathbf{z}=(z_1,z_2)$ that fulfills

$$g(a+z_1,z_2)-g(a-z_1,z_2)-\Delta p=0,$$

where $\Delta p = p_1 - p_2$. When firms quote equal prices, the boundary dividing their market areas is the perpendicular bisector of the segment connecting the locations of the firms.[17] As firm 1 increases its price, the boundary dividing the market areas moves to the right, closer to the location of this firm, and

$$dz_1/d\Delta p = 1/[g_1(a+z_1,z_2)+g_1(a-z_1,z_2)] > 0,$$

where subscripts of g denote partial derivatives. The shape of the boundary for unequal prices determines the intensities of competition at varying distances from the axis of locations of firms.

Consider first the case when the boundary moves parallel to itself as prices change, remaining perpendicular to the segment connecting the locations of the firms. Then, a doubling of the width of the strip doubles the demand,

$$D(\tilde{p},\tilde{p};A)=A\cdot D(\tilde{p},\tilde{p};1). \tag{4}$$

As a firm increases its price, the market boundary moves toward it remaining perpendicular to the axis of locations of the firms. The loss of customers is measured by $D_1(p,\tilde{p};A)$, and it clearly doubles as the width doubles,

$$D_1(\tilde{p},\tilde{p};A)=A\cdot D_1(\tilde{p},\tilde{p};1). \tag{5}$$

Since both the demand and its margin are proportional to the extent of the market, the equilibrium price is independent of the size

---

[15] For a detailed discussion of the marginal consumer in neoclassical theory see William Novshek (1980).

[16] The assumption of inelastic demand has been extensively used in the literature of differentiated products, starting with Hotelling's (1929) seminal paper.

[17] Without loss of generality, let firms 1 and 2 be located at $(a,0)$ and $(-a,0)$, respectively, and quoting prices $p_1=p_2=p$. The boundary is defined by

$$\begin{aligned} & p+f(\|(-a,0)-(z_1,z_2)\|) \\ & \quad = p+f(\|(a,0)-(z_1,z_2)\|) \\ & \quad \Leftrightarrow \|(-a,0)-(z_1,z_2)\| = \|(a,0)-(z_1,z_2)\| \\ & \quad \Leftrightarrow \|(-a-z_1,z_2)\| = \|(a-z_1,z_2)\| \\ & \quad \Leftrightarrow \|(a+z_1,z_2)\| = \|(a-z_1,z_2)\| \\ & \quad \Leftrightarrow a+z_1 = a-z_1 \Leftrightarrow z_1=0, \end{aligned}$$

which defines the perpendicular bisector.

of the market. Combining (4) and (5) implies that

$$D(\tilde{p},\tilde{p};A)+\tilde{p}D_1(\tilde{p},\tilde{p};A)$$
$$=A\left[D(\tilde{p},\tilde{p};1)+\tilde{p}D_1(\tilde{p},\tilde{p};1)\right].$$

Therefore (2′) is equivalent to (2), and

$$(6) \qquad \tilde{p}(1)=\tilde{p}(A)$$

for all A, and in particular for $A=d$. Combining (3) and (6) yields

$$(7) \quad p^C=2\bar{p}(d)>\tilde{p}(d)=\tilde{p}(1)=P^I.$$

Therefore the price of a system under compatibility is higher than under incompatibility.

Since

$$(8) \quad d^2z_1/d\Delta p\,dz_2 = -\left[g_{12}(a+z_1,z_2)+g_{12}(a-z_1,z_2)\right]/\left[g_1(a+z_1,z_2)+g_1(a-z_1,z_2)\right]^2,$$

this case of parallel movement arises when the cross-partial derivative with respect to coordinates is zero: $g_{12}=0$. This is the case when the disutility of distance is linear in the square of the Euclidean distance,

$$g(\mathbf{x},\mathbf{y})=\lambda\left[(x_1-y_1)^2+(x_2-y_2)^2\right],$$

where λ is a positive scalar. Such a specification has been used in one-dimensional settings by Claude D'Aspremont et al. (1979) and Nicholas Economides (1989). The same boundary movement results when the transportation cost function is linear in the "block distance" between points,

$$g(\mathbf{x},\mathbf{y})=\lambda\left[|x_1-y_1|+|x_2-y_2|\right],$$

λ scalar. This transportation cost function is used extensively in urban economics. See Eaton and Lipsey (1980).

Alternatively, the boundary can be concave toward the firm that raises its price above the opponents'. See Figure 3. Let consumer A at (z_1,z_2) be indifferent between buying from either firm. Suppose $p_1>p_2$ so that $z_1>0$. Consider the choice of consumer B located on the vertical through A at distance dz_2 closer to the axis of locations of the firms. When he buys from firm 1, he receives approximately $g_2(z_1-a,z_2)\cdot dz_2$ more utility than the consumer at A. Similarly, when he buys from firm 2, he receives approximately $g_2(z_1+a,z_2)\cdot dz_2$ more utility than the consumer at A. He prefers to buy from firm 1 when $g_2(z_1-a,z_2)\cdot dz_2>g_2(z_1+a,z_2)\cdot dz_2$. This is implied from $g_{12}<0$ because $z_1>0$ and $a>0$. Thus, the boundary is concave toward the higher-priced firm if and only if the cross-partial derivative of the transportation cost function is negative.[18] A boundary that bends more and more toward the axis of location of the firms as the higher-priced firm increases its price implies that, as the higher-priced firm increased its price, more and more consumers are lost far away from the axis than close to it. Thus competition is more intense further away from the axis. This implies lower equilibrium prices as the width of the market increases,[19] that is,

$$\tilde{p}(2A)<\tilde{p}(A).$$

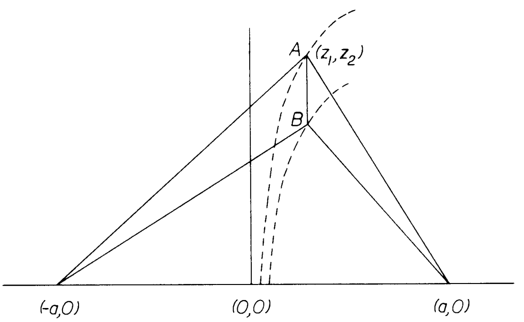

FIGURE 3. THE BOUNDARY DIVIDING THE MARKET AREAS IS CONCAVE TOWARD THE HIGHER-PRICED FIRM

[18]Formally, this is immediate from (8).

[19]With reference to Figure 4, let all other firms except firm j charge $\tilde{p}$. When firm j charges $\tilde{p}$, the

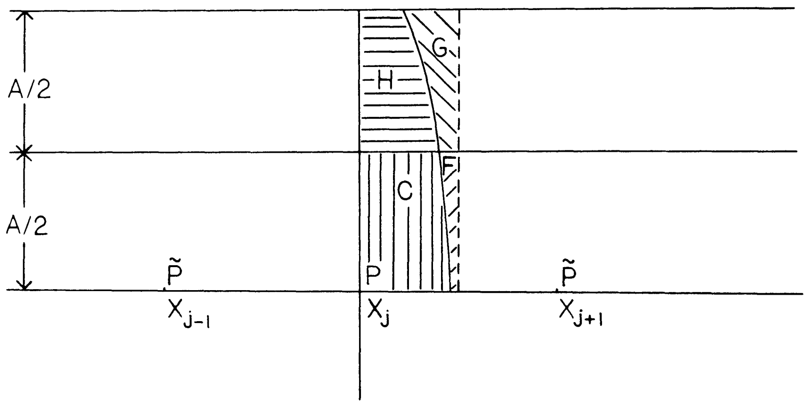

FIGURE 4. MARKET AREAS FOR p LARGER THAN $\tilde{p}$

Since $d<1$, using (3) it follows that

$$(9)\quad p^C = 2\bar{p}(d) > \tilde{p}(d) > \tilde{p}(1) = p^I.$$

Therefore the inequality between p^C, and p^I is stronger there than in the case of parallel boundary movement (equation (7)).

Transportation cost functions in the family

$$g^t(\mathbf{x},\mathbf{y}) = \lambda\left(\sum_{i=1}^{2} |x_i - y_i|^t\right)^{1/t},$$
$$t \in (1,\infty),\ \lambda > 0,$$

which include the Euclidean distance function for $t=2$, $\lambda=1$, fulfill $g_{12}<0$ and therefore result in a boundary that is concave toward the higher-priced firm.[20]

To formally state the results of the last two sections we assume:

ASSUMPTION 1: *The cross-partial derivative of the transportation cost function (de-*

boundary of its market area to the right is vertical to the axis of location at the midpoint of $[x_j, x_{j+1}]$. As firm j increases its price above $\tilde{p}$, let the right boundary shift to the left and become concave toward x_j. Figure 4 shows the boundary for market widths A and $2A$. In this figure, C and $C+H$ are proportional to the demand in markets of width A and $2A$, $C = D(\cdot;A)/2$, $C+H = D(\cdot;2A)/2$. F and $F+G$ represent the margins of the demand in these two markets, $F = |D_1(\cdot;A)|/2$, $G+F = |D_1(\cdot;2A)|/2$. Since the boundary is concave toward the firm with the higher price, the margin grows faster than the demand as the width A doubles. Therefore competition in the outlying strip $[A/2, A]$ is more intense than in the inner strip $[0, A/2]$. It follows that competition in the combined strip $[0, A]$ is more intense than on $[0, A/2]$, and therefore equilibrium prices fall as A doubles. Formally, $H<C$ and

$$G > F \Rightarrow H/C < 1 < G/F$$
$$\Rightarrow (H+C)/C < 2 < (G+F)/F$$
$$\Rightarrow D(\cdot;2A)/D(\cdot;A) < |D_1(\cdot;2A)|/|D_1(\cdot;A)|$$

Using (2′) we derive

$$|D_1(\cdot;2A)|/|D_1(\cdot;A)|$$
$$= [\tilde{p}(A)/\tilde{p}(2A)][D(\cdot;2A)/D(\cdot;A)].$$

Substituting the ratio of the margins and simplifying yields

$$\tilde{p}(2A) < \tilde{p}(A).$$

[20]
$$g_1^t(\mathbf{x},\mathbf{y}) = \lambda\left(\sum_{i=1}^{2} |x_i - y_i|^t\right)^{-1+1/t} |x_1 - y_1|^{t-1},$$

and

$$g_{12}^t(\mathbf{x},\mathbf{y}) = \lambda t(-1+1/t)\left(\sum_{i=1}^{2} |x_i - y_i|^t\right)^{-2+1/t}$$
$$\times |x_1 - y_1|^{t-1}|x_2 - y_2|^{t-1} < 0$$

since $t>1$ implies $-1+1/t<0$. For a detailed discussion of the market boundary when the transportation cost is linear in the Euclidean distance ($t=2$, $\lambda=1$), see Fetter (1924) and Economides (1986).

fined on coordinate differences) *is non-positive.*

This condition is sufficient to ensure that the boundary is concave toward the firm that quotes the higher price (or stays perpendicular to the axis of location of the firms). Throughout this paper, A1, A2, and A3 stand for Assumptions 1, 2, and 3, respectively.

PROPOSITION 1: *When consumers have inelastic demand functions, and the transportation cost function follows Assumption A1, prices are higher for all firms under compatibility.*[21]

B. *General Demand*

We now analyze the effects of doubling the width of the market on the equilibrium price because of the substitution of the differentiated product with the "outside" homogeneous good. Above, we named this the "second margin" of the demand.

We compare equilibrium prices between two markets of widths A and $2A$ under incompatibility. Demand in a market of width A is generated by typical consumers located at $\mathbf{z}=(z_1, z_2)$ with $z_2 \in [0, A/2]$, and by their symmetric counterparts in $[0, -A/2]$. In the following discussion we disregard the latter by utilizing symmetry. In the "upper part" of a market of width $2A$, demand is generated by consumers at $\mathbf{z}$ as above and by consumers located at $\mathbf{z}'=(z_1, z_2+A/2)$. We can write the profit generated by consumers at z as

$$\pi_z = pX(p+g(\mathbf{z}))$$

and the profit generated by consumers at z' as

$$\pi_{z'} = pX(p+g(\mathbf{z}')).$$

Since z' is farther away from the location of the offered system than z is, there exists a nonnegative price $\underline{p}$ such that

$$X(p+g(\mathbf{z}')) = X(p+\underline{p}+g(\mathbf{z})).^{22}$$

Therefore $\pi_{z'}$ can be written as

$$\pi_{z'}(p, \underline{p}) = pX(p+\underline{p}+g(\mathbf{z})).$$

Note that when $\underline{p}=0$, $\pi_{z'}=\pi_z$. *It is as if the marginal cost of goods delivered at* $\mathbf{z}'$ *were higher by* $\underline{p}$ *than of goods delivered at* $\mathbf{z}$. We can now utilize the standard result that "when the demand is concave, increases in the constant marginal cost decrease the differential of the (monopolist's) price over marginal cost." Thus, we assume,

ASSUMPTION 2: *The demand for consumers "residing" at any point of the space is downward sloping and weakly concave.*

Under A2, the price that maximizes π_z is higher than the price that maximizes $\pi_{z'}$.[23]

[21]We note that it is also possible to find transportation cost functions that violate A1 and exhibit $g_{12} > 0$. An example is

$$g(\mathbf{x},\mathbf{y}) = [|x_1 - y_1| + |x_2 - y_2|]^2,$$

the square of the block metric distance, for which $g_{12} = 2 > 0$. For this transportation cost function the result of Proposition 1 may be reversed. As the following example shows, this transportation cost function is not a distance function because it violates the triangle inequality. Take $A=(0,0)$, $B=(0,1)$ and $C=(1,0)$. Then $g(A,B)=1$, $g(B,C)=4$, $g(A,C)=1$, which implies $g(A,B)+g(A,C)=2<4=g(B,C)$, and the triangle inequality fails.

[22]Of course, by its definition, $\underline{p}$ will, in general, be different for different locations of consumers.

[23]Let p^* be defined as the solution of

$$\partial\pi_{z'}/\partial p = X(p^*+\underline{p}+g(\mathbf{z})) + p^*X'(p^*+\underline{p}+g(\mathbf{z}))=0.$$

Then

$$dp^*/d\underline{p} = -\left[X'(p^*+\underline{p}+g(\mathbf{z}))+p^*X''(p^*+\underline{p}+g(\mathbf{z}))\right] \Big/ \left[2X'(p^*+\underline{p}+g(\mathbf{z}))+p^*X''(p^*+\underline{p}+g(\mathbf{z}))\right]$$

which is negative when the demand is (weakly) concave.

Competition is more intense further away from the axis of location of the firms. Since a firm's profits function is the summation of π_z's when the width of the market is A, while it is the summation of π_z's and $\pi_{z'}$'s when the width of the market is $2A$,[24] the equilibrium price will be higher for the smaller width A,

$$\tilde{p}(2A) < \tilde{p}(A).$$

It follows that

$$\tilde{p}(d) \equiv \tilde{p}(1/n) > \tilde{p}(1), \tag{10}$$

and, combined with (3),

$$p^C \equiv 2\bar{p}(d) > \tilde{p}(d) > \tilde{p}(1) \equiv p^I. \tag{11}$$

Therefore, ignoring the effects of the movement of the boundary, and concentrating on the variation of consumption of old customers, under A2 the equilibrium prices are higher in the compatibility regime. We have shown in the previous section that when transportation cost functions follow A1, and when old customers do not change the level of their consumption, then equilibrium prices are higher under compatibility. Combining the effects of the two margins we have:

PROPOSITION 2: *When transportation costs follow A1, and consumers' demand functions follow A2, equilibrium prices are higher in the regime of full compatibility.*

V. Comparison of Profits

We now compare equilibrium profits under the two regimes. Note that the profits functions under symmetry in both regimes can be written as special cases of one function. Let

$$\Psi(p; A) \equiv pD(p, p; A)/A - F.$$

It is immediate that

$$\Pi^C(p/2, p/2, p/2, p/2) = \Psi(p; d),$$

$$\Pi^I(p, p) = \Psi(p; 1).$$

Then the equilibrium profits under compatibility and incompatibility are respectively,

$$\Pi^{C*} = \Psi(p^C; d), \qquad \Pi^{I*} = \Psi(p^I; 1). \tag{12}$$

In the comparison of equilibrium profits, there are two differences to recognize. First, the maximum distance consumers travel under compatibility is significantly lower than under incompatibility. Thus, Ψ is evaluated at d under compatibility, and at $1 > d$ under incompatibility. Second, equilibrium prices are different. The difference in profits is

$$\Pi^{I*} - \Pi^{C*} = \left[\Psi(p^I; 1) - \Psi(p^I; d)\right] - \int_{p^I}^{p^C} \Psi_1(p; d)\, dp. \tag{13}$$

Consider a doubling of the width of the market from A to $2A$ under incompatibility. Because consumers in the new area are located further away from the position of the available product, the "delivered price" is higher for them. Since demand is (weakly) downward slopping, as the market width doubles, the demand expands but does not double. Therefore,

$$D(p, p; d)/d \geq D(p, p; 2d)/(2d) \geq \cdots \geq D(p, p; 1).$$

[24] The profit function in market of width A is

$$\Pi^I(p, \tilde{p}; A) = pD(p, \tilde{p}; A) = 4\int_0^{A/2} \int_0^{\bar{z}_1 - x_j} \pi_z \, dz_1\, dz_2 - F,$$

where $\bar{z}_1 = \bar{z}_1(z_2, p, \tilde{p})$ is the boundary to the right of the market area of firm j defined by

$$p + g(x_j - z_1, z_2) = \tilde{p} + g(x_{j+1} - z_1, z_2).$$

The profit function in a market of width $2A$ can be similarly expressed as

$$\Pi^I(p, \tilde{p}; 2A) = pD(p, \tilde{p}; 2A) = 4\int_0^{A/2} \int_0^{\bar{z}_1 - x_j} (\pi_z + \pi_{z'}) \, dz_1\, dz_2 - F.$$

Hence

$$(14)\quad \Psi(p^I;d) \geq \Psi(p^I;2d) \geq \cdots \geq \Psi(p^I;1) = \Pi^{I*},$$

that is, for fixed prices firms make lower profits as the width of the market expands. The term in brackets in equation (13) is non-positive.

To complete the proof that Π^{I*} is smaller than Π^{C*}, it is sufficient to show that $\Psi(p^I;d)$ is less than $\Psi(p^C;d) = \Pi^{C*}$. Since $p^C > p^I$, and assuming that Ψ is concave (which is the sufficient condition for the uniqueness of the equilibrium and its global stability) it is sufficient to show that $\Psi_1(p^C;d) > 0$. Using (1) we have

$$\Psi_1(p^C;d) = p^C[(n-1)(D_1+D_2)+nD_2]/(2nd),$$

where the subscripts denote partial derivatives. $D_1 + D_2$ is the effect on demand of firm 1 of an equal price increase by all firms. Such an increase leaves the boundaries dividing market areas unaffected. Therefore $D_1 + D_2$ measures the loss of consumers located within the market area of a firm to the "outside" good. Thus $D_1 + D_2 \leq 0$. $D_2 > 0$ is the change in the demand when the opponent increases his price. Since the two terms have opposite signs, the sign of $\Psi_1(p^C;d)$ is ambiguous. $\Psi_1(p^C;d)$ will be positive for a relatively inelastic demand where $D_1 + D_2$ is small and overshadowed by the positive D_2. We assume,

ASSUMPTION 3: *The demand is relatively inelastic so that* $nD_2 \geq -(n-1)(D_1+D_2)$.

Then

$$(15)\quad \int_{p^I}^{p^C} \Psi_1(p;d)\,dp = \Psi(p^C;d) - \Psi(p^I;d) > 0.$$

Combining (14) and (15) yields

$$\Pi^{I*} = \Psi(p^I;1) \leq \Psi(p^I;d) < \Psi(p^C;d) = \Pi^{C*}.$$

Therefore profits will be higher in the regime of compatibility.

Note that condition A3 is by no means necessary. Even when it is violated, it is still possible that $\Psi(p^C;d) - \Psi(p^I;d) = \int_{p^I}^{p^C}\Psi_1(p;d)\,dp > 0$, since by utilizing (2′) we have that $\Psi_1(p^I;d) = p^I D_2 > 0$. Further, the first negative term in (13) could outweigh the worst case when $\Psi(p^C;d) - \Psi(p^I;d) < 0$.

PROPOSITION 3: *Under Assumptions A1, A2, and A3, firms make higher profits under compatibility.*

Thus, we have two very strong results. Under general conditions that are far from pathological, and in the absence of "network" externalities, firms have higher equilibrium prices and profits in the regime of compatibility. Clearly our results can easily be generalized to the analysis of systems that are composed of three or more components.

VI. Free Entry

The analysis this far has been done under the assumption of a fixed number of competing firms. An entry stage can be added as a first stage, and price competition can be viewed as a second stage. In this framework, the fixed cost, F, is paid in the first stage and considered bygone in the second stage. Since for any number of firms, n, profits under incompatibility are lower than under compatibility, free entry will result in a smaller number of active firms under incompatibility:

$$n^I < n^C.$$

This implies that firms would locate their products closer under compatibility:

$$d^C = 1/n^C < 1/n^I = d^I.$$

Even with the number of firms equal, the minimal distance a consumer had to travel (on the average) to an available product location was lower under compatibility, where products were available on a grid rather than on a line. Under free entry, under compatibility, consumers have to travel even less to find an available combination of features.

Prices fall as competing firms come closer in product specification. Since under free entry firms will be located closer under compatibility, the inequality between prices in the two regimes (derived when the number of firms was fixed) will not necessarily be preserved to the free-entry conditions. The following special case shows that under typical assumptions prices will be higher under compatibility even when free entry is allowed.

VII. Special Case: Inelastic Demand and Quadratic Transportation Costs

Let all consumers buy one unit of a system and let the transportation costs be quadratic in the Euclidean distance, $g(\mathbf{a}) = a_1^2 + a_2^2$. This is an important special case that has been discussed in detail in the differentiated products literature.[25] In a market of width A (see Figure 2) the standard demand generated for a system sold at price p_j when other firms charge p is

$$D(p_j, p; A) = A\left(d^2 + p - p_j\right)/d.$$ [26]

Under compatibility $A = d$ and hence

$$D(p_1 + \bar{p}_2, \bar{p}_1 + \bar{p}_2; d) = d^2 + \bar{p}_1 - p_1.$$

Equation (1) yields the equilibrium price,

$$p^C = 2\bar{p} = 2d^2.$$

Under incompatibility $A = 1$ and hence

$$D(p_j, \tilde{p}; 1) = \left(d^2 + \tilde{p} - p_j\right)/d.$$

Equation (2) yields the equilibrium price,

$$p^I = d^2.$$

Thus, when the same number of firms are active, the price of a system under compatibility is twice as high as under incompatibility.

Equilibrium profits under compatibility are $\Pi^{C*} = 2d^3 - F$, while under incompatibility they are $\Pi^{I*} = d^3 - F$. As functions of the number of active firms in the industry, $n = 1/d$, they can be written as $\Pi^{C*}(n) = 2/n^3 - F$ and $\Pi^{I*}(n) = 1/n^3 - F$. At a free-entry equilibrium firms have entered until profits are zero. Thus, there are $n^C = (2/F)^{1/3}$ and $n^I = (1/F)^{1/3}$ active firms under compatibility and incompatibility, respectively. As proved in generality earlier, $n^C > n^I$. The implied equilibrium prices are $P^C(n^C) = 2^{1/3}F^{2/3}$ and $P^I(n^I) = F^{2/3}$. Therefore *under free entry there are approximately* 26 *percent more firms in the regime of compatibility and prices are approximately* 26 *percent higher*.[27]

The comparison between the surplus generated in each regime is not obvious. Since profits are zero in both regimes, it is sufficient to look at consumers' surplus. Under compatibility, for any fixed number of produced goods there is a larger number of available combinations (systems); this tends to make consumers' surplus larger under compatibility because it reduces the distance consumers "travel." Also, under compatibility the number of active firms is larger. This contributes positively to consumers' surplus under compatibility for the same reason. However, prices are higher under compati-

[25] See D'Aspremont et al. (1979) and Economides (1989a).

[26] The boundary between the market areas of firms j and $j+1$ located d apart and quoting prices p_j and p, respectively, is $\bar{z}_1 = (d^2 + p - p_j)/(2d)$. Then the demand for firm j is

$$D(p_j, p; A) = 4\int_0^{A/2}\int_0^{\bar{z}_1} dz_1\, dz_2 = A\left(d^2 + p - p_j\right)/d.$$

[27]

$$(n^C - n^I)/n^I = \left[p^C(n^C) - p^I(n^I)\right]/p^I(n^I) = 2^{1/3} - 1 \simeq 0.2599.$$

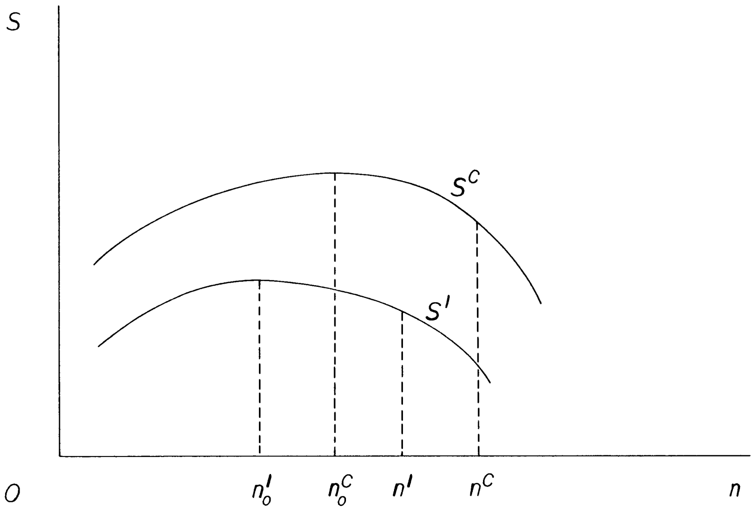

FIGURE 5. TOTAL SURPLUS IN THE TWO REGIMES

bility, and this tends to reduce consumers' surplus in this regime. Thus the direction of the comparison cannot be decided a priori. We now calculate and compare total surplus at the equilibria of the two regimes.

In a market of width A (see Figure 2) the surplus that accrues to consumers who buy a particular system at price p is

$$cs(A,d) = 4\int_0^{d/2}\int_0^{A/2}(k - p - z_1^2 - z_2^2)\,dz_2\,dz_1$$
$$= Ad(k - p - A^2/12 - d^2/12).$$

Under compatibility, the market area of each system is a square of dimensions $d = 1/n$ by $A = 1/n$, and there are n^2 such market areas. Thus total consumers' surplus under compatibility is

$$CS^C(n) = n^2 \cdot cs(1/n, 1/n)$$
$$= k - p - 1/6n^2.$$

Profits of a typical firm are $\Pi(n) = p/n - F$, so that total surplus is

$$(16) \quad S^C(n) \equiv CS^C(n) + n\Pi(n)$$
$$= k - nF - 1/6n^2.$$

Under incompatibility, the market area for each system is a parallelogram of dimensions $d = 1/n$ by $A = 1$, and there are n such market areas. Thus consumers' surplus is

$$CS^I(n) = n \cdot cs(1, 1/n)$$
$$= k - p - 1/12 - 1/12n^2.$$

Profits of a typical firm are $\Pi(n) = p/n - F$, so that total surplus under incompatibility is

$$(17) \quad S^I(n) \equiv CS^I(n) + n\Pi(n)$$
$$= k - nF - 1/12 - 1/12n^2.$$

A direct comparison of (16) and (17) reveals that $S^I(n) < S^C(n)$, as expected, since for any fixed number of firms there are on the average more product combinations available to the consumers under compatibility and they travel shorter distances. However, the comparison of actual realized surplus at equilibrium can go either way since the equilibrium numbers of firms differ, $n^C \neq n^I$. See Figure 5. Substituting n^C and n^I in (16) and (17) we derive the realized total surplus at the two equilibria,

$$S^C(n^C) = k - (13/12)2^{1/3}F^{2/3},$$
$$S^I(n^I) = k - (13/12)F^{2/3} - 1/12.$$

Total surplus is higher under compatibility iff

$$F<1/\left[13(2^{1/3}-1)\right]^{3/2}\simeq 0.04738,$$

which corresponds to $n^I \geq 3$ or $n^C \geq 6$. Thus, *the compatibility free-entry equilibrium is socially preferred to the free-entry equilibrium under incompatibility if and only if fixed costs are small*, or equivalently if and only if the number of active firms at the compatibility equilibrium is larger or equal to six.

A planner able to choose the number of active firms will always choose compatibility because it makes available all the hybrid systems to the great benefit of consumers without loss to the collective interests of the producers. Total surplus under compatibility (equation (16)) is maximized at $n_0^C = (1/3F)^{1/3}$ active firms, a number that is smaller than both the free-entry equilibrium number of firms under compatibility, $n^C = (2/F)^{1/3}$, and under incompatibility, $n^I = (1/F)^{1/3}$. See Figure 5. *We observe an overabundance of varieties in both regimes compared to optimality.* This result is in the same vein with the traditional result in markets for substitute products differentiated by one of their characteristics as in Steve Salop (1979) and Economides (1989a).

VIII. Concluding Remarks

Two factors were crucial in establishing our results. First, the elasticity of the (residual) demand the firm faces when it competes in components is lower than when it competes in systems. Second, the extent of the market is larger under incompatibility. A direct consequence of the first factor is the fact that prices are higher under compatibility. This result is strengthened by the difference of the extents of the markets when the demand function at any "location" is concave. The first factor is the fundamental driving force of the result. The intuition behind it is not confined to the specific model of differentiated products, and can be used in any situation where complementary goods may be produced either independently by two or more oligopolists or by the same oligopolist.

The analysis of this paper points to the desirability from the point of view of the firm of a regime of full compatibility. Arguments pertaining to positive consumption ("network") externalities advanced by Katz and Shapiro (1985) and others result in the same conclusions, albeit for different reasons. Why, then do we sometimes observe products sold as systems that are not decomposable into components (such as the original *Macintosh* by *Apple*), or being composed of incompatible components (VHS vs. Beta videocassettes and players)? The model presented here assumes that the same technology of production of both components is available to all firms, and that there are no cost savings from the combination of both components as a system. Our results do not necessarily carry over to a situation where one firm has a technological advantage in the production of one of the components or a strategic advantage in the game. Further, when the technological advantage comes from proprietary information, a system-producing firm may be unwilling to disclose the specifications that could make the linkage of two components feasible, because such a disclosure could make some of the proprietary information public and lead to early imitation. This reasoning may explain the undecomposable design of the original *Macintosh* as well as the absence of close imitations (clones). In an asymmetric setting a firm can sometimes introduce a new incompatible product attempting to establish a new industry "standard," and as a consequence establish itself, at least in perception, as the leader of the industry. It seems that attempts to establish leadership in the field were primary considerations in the introduction of competing standards in the video recorders industry. The analysis of compatibility under asymmetric strategic and technological conditions is still undeveloped.

Our model compares two extreme situations of full compatibility and full incompatibility. Since we do not analyze situations where some firms produce compatible components and some do not, we cannot claim that in general firms acting noncooperatively will choose mutual compatibility. However, in the special case of duopoly, each firm has the ability to force incompatibility in the industry. Then, the two-stage game, where firms choose between compatibility and incompatibility in the first stage and choose prices in the second stage, has the compatibility regime as a unique perfect equilibrium. This is an immediate consequence of the fact that profits are higher under compatibility.

Recent work (Economides, 1989b) has shown that this tendency toward full compatibility holds even when firms have the opportunity to vary the *degree* of compatibility of their components with the components of the competitor. The degree of compatibility is measured by the cost of an adapter that is required to make a hybrid system function. Economides (1989b) shows that in a two-stage duopoly game, where the degree of compatibility is chosen in the first stage, and prices are chosen in the second stage, the perfect equilibrium is at full compatibility, that is, at zero-adapter's cost provided that the demand for hybrids and single-producer goods are of the same size. If demand for hybrids is low, or if demand for systems that contain at least one component made by firm 1 is low, then at least one firm chooses incompatability. Thus, symmetry of the demand system is crucial for compatibility.

REFERENCES

D'Aspremont, Claude, Jaskold-Gabszewicz, Jean and Thisse, Jacques-François, "On Hotelling's 'Stability in Competition'," *Econometrica*, September 1979, *47*, 1145–50.

Bergstrom, Theodore, "Cournot Equilibrium in Factor Markets," mimeo., University of Michigan, 1988.

Cournot, Augustin, *Researches into the Mathematical Principles of the Theory of Wealth*, Nathaniel Bacon, trans., New York: Macmillan, 1927.

Discussion Paper No. 157, Center for Economic Policy Research, Stanford University, 1989.

Eaton, B. Curtis and Lipsey, Richard, "The Block Metric and the Law of Markets," *Journal of Urban Economics*, May 1980, *7*, 337–47.

Economides, Nicholas, "Nash Equilibrium Existence in Duopoly with Products Defined by Two Characteristics," *Rand Journal of Economics*, Autumn 1986, *17* 431–39.

______, (1989a) "Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets," *Journal of Economic Theory*, February 1989, *47*, 178–94.

______, (1989b) "Variable Compatibility Without Network Externalities," mimeo., 1988.

Fetter, Frank, "The Economic Law of Market Areas," *Quarterly Journal of Economics*, May 1924, *38*, 520–29.

Katz, Michael and Shapiro, Carl, "Network Externalities, Competition, and Compatibility," *American Economic Review*, June 1985, *75*, 424–40.

Hotelling, Harold, "Stability in Competition," *Economic Journal*, March 1929, *39*, 41–57.

Matutes, Carmen and Regibeau, Pierre, "Mix and Match: Product Compatibility Without Network Externalities," *Rand Journal of Economics*, Summer 1988, *19*, 221–34.

Novshek, William, "Marginal Consumer in Neoclassical Economics," mimeo., 1980.

Salop, Steve, "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, Spring 1979, *10*, 141–56.

Sonnenschein, Hugo, "The Dual of Duopoly Is Complementary Monopoly: or, Two of Cournot's Theories Are One," *Journal of Political Economy*, March-April 1968, *76*, 316–18.