International Journal of Industrial Organization 11 (1993) 219–237. North-Holland

Quantity leadership and social inefficiency

Nicholas Economides*

Stern School of Business, New York University, New York, NY10012–1126, USA

Final version received April 1992

A game of simultaneous free entry and sequential output choices is analyzed. Firms enter simultaneously in stage 1 by paying a fixed cost, and they choose output levels sequentially in subsequent stages. At the subgame-perfect equilibrium of the game, the production level of a firm is decreasing with the order of the firm in the decision-making. The firm that is the last to choose output produces the same amount as a typical firm in the standard symmetric simultaneous-moves Cournot game. Moreover, industry output and market price are identical in the sequential and the symmetric Cournot games. It follows that in the sequential game there are *fewer* active firms and *higher* total surplus than in the standard symmetric Cournot game. Strategic asymmetry results in higher concentration and higher total surplus without an increase in price!

1. Introduction

In the context of quantity-setting oligopoly, entry deterrence has been recently discussed by Dixit (1979), Nti and Shubik (1981), Boyer and Moreaux (1986), Gilbert (1986), Gilbert and Vives (1986), Eaton and Ware (1987), Vives (1988), Basu and Singh (1990), and Robson (1990), among others. These authors focus on the effect of quantity and investment decisions of incumbents on the *entry* decision of a potential entrant. In the context of the models mentioned, it is important that entry occurs sequentially and implies the expenditure of a fixed cost. In Dixit (1979), for example, the decisions of firm 2 whether or not to enter and what quantity to produce are taken simultaneously in stage 2, which follows stage 1 where firm 1 chooses entry and output simultaneously. Then the best response function of a later-acting firm to an increase in the output of an earlier-acting firm is, for a

Correspondence to: N. Economides, Suite 7-89, Stern School of Business, Management Education Center, 44 West 4th Street, New York, NY 10012–1126, USA.

*I thank Tina Fine, Frank Hahn, Andreu Mas-Colell, Xavier Vives, and especially two anonymous referees and Larry White for useful suggestions on an earlier draft. Partial financial support from the National Science Foundation is gratefully acknowledged.

0167–7187/93/$06.00

range of values, discontinuous. That is, a slight increase in the output of firm i results in a close-down response by firm $i+1$, although firm $i+1$ was not initially producing an infinitesimal output. Thus, when entry and output of firm $i+1$ are chosen simultaneously, the strategic asymmetry embodied in quantity leadership implies a discontinuous output response by a follower to the action of a leader.

In another train of thought [von Stackelberg (1934)], the ability of a 'leader' to act first has been taken to have an effect on output decisions of 'followers', but not on their presence in the market. This is formalized in the present paper through a separation of the entry and output choice decisions. The decision on entry is taken at an early stage, before any firm chooses output. Thus, a model of sequential leadership is analyzed where firms that choose output levels earlier in the sequence of decisions are able to influence the output choices of those that act later, but are not able to throw them out of business. In our framework, at the time of output decisions, the entry stage has been completed and the number and identity of competitors cannot be altered. Thus, entry deterrence of the form described in the articles above cannot occur.

The models mentioned above allow firms to make a strong commitment in both the presence in the market and in output choice. In contrast, the standard Cournot model does not allow any commitments at all. The model of the present paper falls in between. It allows firms to use strategically their commitment in output, but does not allow them to use strategically their decisions regarding their presence in the market.[1]

In refinements of the structure–conduct–performance paradigm, the number of competitors in an industry is generally considered an element of *structure*, while the levels of output and price are elements of *conduct*. In the emerging formalization of the structure–conduct–performance paradigm in the form of a multi-stage game, the decisions affecting directly structural elements are chosen in earlier stages of the game while the decisions on conduct are chosen in later stages of the game. In a multi-stage game, the earlier stages are reserved for the variables that are most difficult to vary in the short run. Thus, the late stages correspond to the short run and the early stages correspond to the long run. For example, in the standard model of variety-differentiated products [Salop (1979), Economides (1989)], entry is chosen in stage 1, locations (varieties) are chosen in stage 2, and prices are chosen in stage 3. In short, there is a tradition of separating the entry

[1] Another possible game in which firms enter sequentially but choose quantities simultaneously results in the free entry equilibrium of the standard Cournot game. Thus, in a model of homogeneous goods, the ability to enter early, without the ability to commit in quantity, is ineffective. However, in a model of *differentiated* products, sequential entry and/or sequential product choice with simultaneous pricing results in equilibria that differ significantly from the simultaneous entry ones.

decisions from the decisions of quantity choices, as in fact we shall do in this paper.

It is common for firms to first decide if they are going to participate in a certain industry, and later decide the specific way in which they are going to compete and price their output. Output decisions are typically taken often and there is significant flexibility associated with such choices. Entry decisions, by their very nature, are rare. Entry involves actions that are not typically taken in the course of day-to-day business. In our setting, this puts the entry decision before the quantity decision. Thus, in the refined model of the structure–conduct–performance paradigm that we have in mind, entry happens in the long run, capacity choice takes place in the medium run, and pricing choices take place in the short run. In quantity-setting models, capacity and pricing decisions have traditionally been treated as taking place simultaneously, with recent exceptions.[2,3]

Some examples of Stackelberg equilibria (with a fixed number of firms) exhibit higher social welfare than the Cournot equilibrium.[4] This result is sometimes reversed in the literature mentioned above, where the leader(s) use his (their) commitment on quantity to restrict entry. But if the strategic asymmetry is modeled so that the output decisions of a leader do not have a *direct* effect on entry (i.e. do not create a discontinuous response from the followers), then the intuition of von Stackelberg, that leadership can be socially beneficial, can go through. In fact it is shown in this paper that a sequential leadership equilibrium Pareto dominates the Cournot equilibrium despite the fact that it typically implies a much higher concentration index.

Our model has two phases. The first phase has one stage in which firms enter simultaneously incurring a fixed cost F. They then bid among themselves for a position in the order they will choose production levels. As the outcome of bidding, they are assigned an order and they are labelled $i=1,\ldots,n$.[5] In the second phase, firms choose quantities sequentially, considering the entry cost F as bygone. The second phase has n stages. In stage i, a choice of quantity of production, q_i, is made by the ith firm. In

[2]Kreps and Scheinkman (1983).

[3]In a particular market, the empirical evidence on the relative flexibility of the various decision variables at each time frame will suggest the appropriate game structure of the model that describes best the workings of that market. For example, empirical evidence will make it clear if quantity choice happens simultaneously with an entry decision or not.

[4]See Daughety (1990) and Robson (1990).

[5]The bidding mechanism that assigns order to the firms is not discussed in detail. For example, firms may bid among themselves for factors of production. In a typical industry there are specific assets and resources (e.g. trained personnel) that are necessary for production and in short supply. Firms will tend to compete for these resources. See Zink (1990). Other mechanisms may also be used. For example, an auctioneer can collect all the rents the firms would realize by auctioning the positions. The way in which the order of action is assigned is of no great consequence, as long as no resources are wasted in the process, and the firm that takes the last position does not pay a positive price for getting this position.

making quantity choices, each firm considers all decisions of prior stages as given and beyond its control. Thus, firm k acts as a (quantity) follower to all firms of index $j<k$ and considers their output choices as given. However, each firm k realizes the influence of its output choice on the subsequent decisions of firms of higher index $j>k$ that are made in later stages. Thus, each firm is a (quantity) leader with respect to firms of higher index. We seek subgame perfect equilibria.

It is important to understand that this model of sequential leadership does not describe *sequential entry*. Here entry is simultaneous. At the juncture of output choice (second phase) the number of active firms and their order is already known and given. Therefore *output choices cannot be used to prevent a potential entrant from entering*.

The equilibrium of sequential leadership is compared with the symmetric equilibrium of the standard Cournot game with free entry. The Cournot game has entry in the first stage and simultaneous quantity choice in the second stage. It is shown that the industry output of the sequential leadership model is equal to the industry output of the standard Cournot model of simultaneous output choices! Therefore the two game structures result in the same price and consumers' surplus. On the producers' side, however, there are significant differences between the resulting market structures. The sequential leadership game results in a market structure with significant inequality in the production levels of active firms. An earlier-acting firm has a strategic advantage, which it exploits by producing more than a later-acting firm. Thus, outputs in sequential leadership are inversely related to their order of action. It is shown that *the last-acting firm in sequential leadership produces as much as a typical firm in the Cournot game*. Since in sequential leadership earlier-acting firms produce higher outputs while total output is the same as in symmetric Cournot, it follows that the number of active firms in sequential leadership is smaller than in the standard Cournot model. Thus, in sequential leadership there are fewer active firms than in Cournot, and they exhibit significant inequalities in their output levels. While price and consumers' surplus are the same in both market structures, sequential leadership results in positive total profits and therefore higher total surplus than the Cournot model. Thus, we have the notable result that *a market structure of significant inequality and a smaller number of firms achieves the same price and consumers' surplus but higher total surplus than a market structure of total equality*. This is the result of more efficient exploitation of the increasing returns technology in the sequential leadership market structure.[6]

[6]Efficiency gains from asymmetric games have been demonstrated by Saloner (1987) and Daughety (1990). In particular, Daughety (1990) has a similar model to the one described here. There are two stages with m firms acting in stage 1 and n firms acting in stage 2. All earlier-acting firms are leaders with respect to later-acting firms, but Cournot oligopolists with respect

The rest of the paper is organized as follows. Section 2 presents the general results. Section 3 presents an important special case of linear demand and constant marginal cost, noting the significant differences in Herfindahl indices and total surpluses across the two market structures. In section 4 we summarize our results and discuss extensions and possibilities of further research.

2. The model

We make the following standard assumptions on the demand and cost functions:

Assumption A1. All firms have the same technology represented by cost function $F+C(q)$, where $C(0)=0$, $C'(q)\geqq 0$, $C''(q)\geqq 0$ and $F>0$.

Assumption A2. Industry demand is downward sloping and weakly concave, $P'(Q)<0$ and $P''(Q)\leqq 0$.

These assumptions guarantee that the maximization problem is concave and that the equilibrium exists.

Let a subgame-perfect equilibrium of the game of sequential leadership (S.L.E.) described above be a number of active firms n_s and their corresponding outputs $(q_1^s,\ldots,q_{n_s}^s)$.

The standard Cournot game with free entry is also defined to enable comparisons of its outcome with the one of the sequential leadership game. The standard Cournot game with free entry has two stages. In stage 1, firms enter simultaneously incurring a fixed cost F. The n entrants of the first stage choose quantities simultaneously in the second stage. A *Cournot equilibrium* (C.E.) consisting of n_c active firms and $(q_1^c,\ldots,q_{n_c}^c)$ corresponding output levels is a subgame-perfect equilibrium of this game structure.[7,8]

The structure of our argument on the comparison of the sequential with the Cournot equilibrium follows. We first prove that the *last* firm in the sequential game produces the *same* amount as a *typical* firm in the Cournot game. Secondly, we show the total outputs in the sequential and Cournot games are equal. Both results appear in Theorem 1. Thirdly, we show that

to firms that act in the same period. He finds that concentration may be uncorrelated with social welfare. The significant difference between our model and his is the fixed cost assumed in our cost function and its consequences. In particular, in our model, earlier-acting firms are able to use the technology more efficiently thereby improving the overall efficiency of the sequential leadership model in comparison with the Cournot model.

[7]Attention will be confined to symmetric Cournot equilibria (C.E.) See Novshek (1984) for a discussion of the existence and algorithms of calculation of asymmetric Cournot equilibria.

[8]A game where firms enter as well as choose production levels simultaneously gives the same equilibrium as the Cournot game.

output decreases in the order of action in the sequential game (Theorem 2). Combined with the earlier results, it follows that there is a smaller number of active firms at the equilibrium of the sequential game (Corollary 1). Since the last firm in the sequential game makes zero profits, earlier firms make positive profits (Theorem 3). Since consumers are equally well off in the two games, the sequential game is Pareto superior to the Cournot game (Theorem 4) and results in a higher surplus (Corollary 2).

We start with the crucial argument that the last firm in the sequential leadership game produces the same amount as the typical firm in the Cournot game. This argument runs as follows. First we note that any given amount produced by other firms, y, the last firm in sequential leadership and the typical firm in Cournot respond by producing the same amount of output $q^*(y)$, and realize the same amount of profits. Free entry sets profits to zero, and determines uniquely y and $Q=q^*(y)+y$ as the total equilibrium output in the market in both models. Monotonicity of Q in y and of q^* in y implies the same output and profits for the last firm in the sequential game and for the typical firm in the Cournot game.

We first characterize the industry output and the output of the last firm in the sequential game. In the last stage of the game, the number of active firms, n_s, their order of action, and the output of all other firms except the last one have been already determined. Consider the output decision of the firm that acts last. Let Q denote the total industry output, and let y be the output of all firms other than the last firm, n_s, so that $Q=y+q_{n_s}$. The profit function of the last firm,

$$\Pi(q_{n_s}, y, F)=q_{n_s}P(q_{n_s}+y)-C(q_{n_s})-F, \tag{1}$$

is maximized with respect to q_{n_s} at $q^*(y)$, the solution of

$$P(q^*+y)+q^*P'(q^*+y)-C'(q^*)=0. \tag{2}$$

Note that q^* is monotonically decreasing in y^*.[9] As expected in quantity games, the last firm, n_s, responds to an increase of output of previously-acting firms, y, through a cut in its own output, q^*. Total output, $Q(y)=q^*(y)+y$, is monotonically increasing in y,[10] as an increase of production by previously-acting firms is not totally absorbed by the decrease of the output of the last firm.

Under the condition of free entry in the first phase of the game, the realized profits of the last active firm are zero:[11]

[9] $dq^*/dy = -[\partial^2\Pi_{n_s}/\partial q_{n_s}\partial y]/[\partial^2\Pi_{n_s}/\partial q_{n_s}^2] = -[q^*P''(q^*+y)+P'(q^*+y)]/[q^*P''(q^*+y)+2P'(q^*+y)-C''(q^*)]<0$ because both numerator and denominator are negative.

[10] $dQ/dy=1+dq^*/dy=[P'(q^*+y)-C''(q^*)]/[q^*P''(q^*+y)+2P'(q^*+y)-C''(q^*)]>0$ since both numerator and denominator are negative.

[11] Of course, this ignores integer constraints. Otherwise profits could be slightly positive.

$$\Pi(q^*(y), y, F) = 0. \tag{3}$$

It is easily shown that the realized profits of the last firm are monotonically decreasing in the total output of the other firms y and in setup cost F. It follows that, for each level of fixed cost F, there exists a unique total output of previously-acting firms, $y = y_z(F)$, such that the last firm realizes zero profits when it optimally responds to this output. $y_z(F)$ is found as a solution of eq. (3). Thus, at a subgame-perfect equilibrium, for a given fixed cost F, there exists a unique output of other-than-last firms, $y_z(F)$, a unique output of the last firm, $q_{n_s} = q^*(y_z(F))$, and a unique total output, $Q(F) = y_z(F) + q^*(y_z(F))$, that are consistent with the last firm optimizing and realizing zero profits.

Looking now at the standard symmetric Cournot game we note that the decision problem of a *typical firm* is identical to the decision problem of the *last firm* of the sequential game. Given an amount y produced by all other firms, firm i in the Cournot game maximizes

$$\Pi(q_i, y, F) = q_i P(q_i + y) - C(q_i) - F, \tag{1'}$$

which is a re-writing of eq. (1) with the index n_s substituted by i.[12] Given y, profits are maximized at $q_i^c = q^*(y)$, the solution of eq. (2). As before, total output is $Q(y) = q^*(y) + y$. At the free entry Cournot equilibrium, profits of a typical firm are zero, i.e.

$$\Pi(q^*(y), y, F) = 0, \tag{3'}$$

which is identical to eq. (3). The same fixed cost F implies the same level of total output of other firms (consistent with optimization) in the two games. Therefore, the output of the typical firm at the standard C.E. is equal to the output of the last firm at the sequential leadership equilibrium (S.L.E.),

$$q_i^c = q^*(y_z(F)) = q_{n_s}. \tag{4}$$

It immediately follows that, for the same fixed cost F, the sequential leadership and the Cournot games have the same equilibrium industry output,

$$Q(y_z(F)) = q^*(y_z(F)) + y(F),$$

and the same market price, $P(Q(F))$.

Theorem 1. The equilibrium industry output is the same in the game of

[12] For the moment, we do not know if there is any relationship between the quantity y of this equation and the quantity y of eq. (1). That is, y here should be thought for the moment as a free variable that is not attached to a particular level.

sequential output decisions as in the symmetric free entry Cournot game. The output of the last firm of the sequential game is equal to the output of a typical firm of the symmetric Cournot game.

Since industry output is the same in the market of sequential leadership and in the Cournot market, concentration indices (such as the Lerner index) that are based on total output or price will not reveal any difference between the two markets. This result holds despite the fact that these markets exhibit significant differences in concentration and profits, as we show next.

At the S.L.E. there is significant inequality in the production levels of firms. As we have shown, the last firm in sequential leadership has the same output as a typical firm in standard Cournot. Earlier-acting firms in sequential leadership will produce more, making use of the strategic advantages of their position. Consider the output decision of the penultimate firm, n_s-1, in the sequential game. This firm is aware of the influence of its output choice q_{n_s-1} on the last firm's output. This influence is precisely

$$\mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1}=\mathrm{d}q_{n_s}/\mathrm{d}y=-(P'+q^*P'')/(2P'+q^*P''-C'')<0. \tag{5}$$

An increase of the output of firm n_s-1 precipitates a decrease in the output of firm n_s. Taking into account its strategic advantage, firm n_s-1 produces more output than firm n_s. This is seen through an analysis of its first-order condition:

$$\partial\Pi_{n_s-1}(q_{n_s-1},y)/\partial q_{n_s-1}=P(q_{n_s-1}+y)+q_{n_s-1}P'(q_{n_s-1}+y)$$
$$\times(1+\mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1})-C'(q_{n_s-1})=0. \tag{6}$$

Evaluating $\partial\Pi_{n_s-1}/\partial q_{n_s-1}$ at $q_{n_s-1}=q^*$ and using eq. (2) yields

$$\partial\Pi_{n_s-1}(q^*,y)/\partial q_{n_s-1}=q^*P'(q^*+y)\,[\mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1}]>0,$$

which is negative since $\mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1}<0$ from eq. (5), and the demand is downward-sloping, $P'<0$. Since the profit function of firm n_s-1 is concave in its own production, it immediately follows that $q^s_{n_s-1}>q^*=q^s_{n_s}$, i.e. the penultimate firm produces more than the last firm.

Using the same technique, we show in the appendix that the output of firm n_s-2 is higher than the output of firm n_s-1, $q^s_{n_s-2}>q^s_{n_s-1}$. Similarly, it can be shown that output of firms that act earlier is even larger, with the first firm producing the highest level of output,

$$q^s_1>q^s_2>\cdots>q^s_{n_s-2}>q^s_{n_s-1}>q^s_{n_s}.$$

Theorem 2. At the S.L.E., a firm's output level varies inversely with its order in the chain of decisions.

Since (total) industry output is equal in the Cournot and in the sequential leadership game, while all firms except the last one produce higher outputs at the sequential leadership game, it follows that there are fewer active firms at the sequential leadership equilibrium rather than in standard Cournot.

Corollary 1. There are fewer active firms at the S.L.E. than at the C.E.

At equilibrium, the last firm has zero profits. Earlier-acting firms have positive profits. We can compare the profits of firms j and $j-1$, where $j \in \{2, \ldots, n_s\}$. Firm $j-1$ produces more than firm j and enjoys the same price as firm j. Furthermore, total output of all other firms is smaller for firm j than for firm $j-1$. Both of these factors contribute to higher profits for firm j.

Formally, through the use of eq. (1), equilibrium profits can be written as a function of the own output choice, the total production of all other firms, and the fixed cost:

$$\Pi^s_{j-1} = \Pi(q^s_{j-1}; y^s_{j-1}, F), \qquad \Pi^s_j = \Pi(q^s_j; y^s_j, F),$$

where

$$y^s_{j-1} = Q^s - q^s_{j-1}, \qquad y^s_j = Q^s - q^s_j, \qquad Q^s = \sum_{i=1}^{n_s} q^s_i.$$

Now, $q^s_{j-1} > q^s_j$ implies $y^s_j > y^s_{j-1}$ by their definition above. Since each firm's profits are decreasing in the output of other firms, $\partial\Pi/\partial y = qP' < 0$, it follows that

$$\Pi(q^s_j; y^s_{j-1}, F) > \Pi(q^s_j; y^s_j, F) = \Pi^s_j. \tag{7}$$

Since quantity q^s_{j-1} maximizes the profits of firm $j-1$, $\Pi(q, y^s_{j-1}, F)$, with respect to q, we have

$$\Pi^s_{j-1} = \Pi(q^s_{j-1}; y^s_{j-1}, F) \geqq \Pi(q^s_j; y^s_{j-1}, F). \tag{8}$$

Combining (7) and (8) it follows that firm j realizes lower profits than the one preceding it, $\Pi^s_{j-1} > \Pi^s_j$. This is true for any j, $2 \leqq j \leqq n_s$, so that

$$\Pi^s_1 > \Pi^s_2 > \cdots > \Pi^s_{n^s} = 0.$$

We have shown,

Theorem 3. *Equilibrium profits decrease with a firm's position in the chain of decisions. Except for the last firm that makes zero profits, all firms make positive profits.*

Next we compare social welfare at the equilibrium of the sequential leadership game with that of the symmetric Cournot game. Each consumer is equally well-off in both games because both yield the same equilibrium price. All active producers except the last one make positive profits at the equilibrium of the sequential leadership game, and thus are better off than at the C.E. where all had zero profits. The last active producer at the equilibrium of the sequential game and all the inactive producers at that equilibrium realize zero profits and are therefore equally well-off as at the C.E. Therefore the outcome of the sequential game is Pareto superior to the Cournot outcome.

Theorem 4. *The equilibrium of the sequential leadership game is Pareto-superior to the symmetric equilibrium of the Cournot game with free entry.*

Total consumers' surplus is equal in the two games since they result in equal industry outputs and prices. At the sequential leadership equilibrium, as noted above, total industry profits are positive, while they are zero at the Cournot equilibrium. It follows that,

Corollary 2. *Total surplus is higher at the sequential leadership game than at the symmetric Cournot game.*

As expected, market concentration is positively correlated with positive profits. But, in this framework, *market concentration and profits are also positively correlated with social welfare.* This is not surprising because the increase in profits is achieved without a decrease of output that would create a 'deadweight loss'. The welfare increase arises from the more efficient utilization of the production technology. Social welfare increases because of savings of fixed costs of firms that were active at the C.E. but are inactive at the sequential equilibrium. These savings may be tempered by variable cost inefficiencies that may arise from the asymmetric distribution of production at the sequential equilibrium if marginal cost is increasing. Of course, the sequential game cannot achieve the 'first best' because its equilibrium industry output, being equal to the Cournot output, differs from the competitive one that is achieved with price equal to marginal cost and a subsidy of F given to the single operating firm.

In the next section we present an example of an industry of linear demand and constant marginal cost. We calculate the sequential output decisions and the C.E. and compare them.

3. An example

In the following example we assume linear demand and constant marginal cost c. The units can be normalized so that the industry demand function is $P=A-Q$. We can substitute $p=P-c$, $a=A-c$, so that $p=a-Q$. We first characterize the n-firm equilibrium of the second phase. In the appendix we prove the following theorem.

Theorem 5. For linear demand and constant marginal cost, outputs form a geometric sequence in the sequential leadership game, with the first firm producing as much as a monopolist would have produced![13]

Firm i produces

$$q_i^{\mathrm{m}}=a/2^i. \tag{9}$$

Total output and price at an n-firm equilibrium are

$$Q^{\mathrm{s}}=\sum_{i=1}^{n} q_i^{\mathrm{m}}=a\sum_{i=1}^{n} 2^{-i}=a(1-2^{-n}), \qquad p^{\mathrm{s}}=a/2^n, \qquad P^{\mathrm{s}}=c+a/2^n. \tag{10}$$

It follows that equilibrium profits for firm i and industry profits of phase 2 are

$$\Pi_i^{\mathrm{s}}=a^2/2^{n+i}-F, \qquad \sum_{i=1}^{n} \Pi_i^{\mathrm{s}}=a^2(2^{-n}-2^{-2n})-nF \tag{11}$$

respectively. Since the last firm at an n_{s}-firm equilibrium makes zero profits, $a^2/2^{2n_{\mathrm{s}}}=F$, or equivalently,

$$2^{n_{\mathrm{s}}}=a/\sqrt{F} \quad \text{or} \quad n^{\mathrm{s}}=[\log a-(\log F)/2]/\log 2. \tag{12}$$

This equilibrium exists if $n^{\mathrm{s}}\geqq 1 \Leftrightarrow a\geqq 2\sqrt{F}$. Substituting in (9)–(11) we derive the full equilibrium of the sequential choice game:

$$q_i^{\mathrm{s}}=a/2^i, \qquad Q^{\mathrm{s}}=a-\sqrt{F}, \qquad P^{\mathrm{s}}=c+\sqrt{F}, \qquad \Pi_i^{\mathrm{s}}=a\sqrt{F}/2^i-F. \tag{13}$$

Profits can also be written as $\Pi_i^{\mathrm{s}}=F(2^{n_{\mathrm{s}}-i}-1)$. From this formula it is immediate that $\Pi_{n_{\mathrm{s}}}^{\mathrm{s}}=0$, $\Pi_{n_{\mathrm{s}}-1}^{\mathrm{s}}=F$, $\Pi_{n_{\mathrm{s}}-2}^{\mathrm{s}}=3F$, $\Pi_{n_{\mathrm{s}}-3}^{\mathrm{s}}=7F$, etc. Total profits are

[13]This result is also proved in Boyer and Moreaux (1986). Gilbert (1986) also quotes this result.

$$\sum_{i=1}^{n_s} \Pi_i^s = a\sqrt{F} - (n_s+1)F > 0,$$

where n_s is given by eq. (12) above. Consumers' surplus is $CS = (a - \sqrt{F})^2/2$. Total surplus in sequential leadership is

$$S_s = CS + \sum \Pi_i^s = a^2/2 - F(1+2n_s)/2$$

$$= a^2/2 - F\{1/2 + [\log a - (\log F)/2]/\log 2\}.$$

In contrast, the n-firm symmetric Cournot equilibrium is

$$q_i^c = a/(n+1), \quad Q^c = na/(n+1), \quad p^c = a/(n+1), \quad P^c = c + a/(n+1). \tag{14}$$

Individual firm and industry profits are

$$\Pi_i^c(n) = a^2/(n+1)^2 - F, \qquad \sum_{i=1}^{n} \Pi_i^c = na^2/(n+1)^2 - nF. \tag{15}$$

Solving $\Pi_i^c(n_c) = 0$ yields the number of active firms, n_c:

$$n_c = a/\sqrt{F} - 1. \tag{16}$$

Substituting in (14) we derive the full equilibrium of the symmetric Cournot game:

$$q_i^c = \sqrt{F}, \qquad Q^c = a - \sqrt{F}, \qquad P^c = c + \sqrt{F}, \qquad \Pi_i^c = 0. \tag{17}$$

Since profits are zero, total surplus for the Cournot case is just the consumers surplus:

$$S_c = a^2/2 + F/2 - a\sqrt{F}.$$

Before starting the comparisons, note that in optimality there is only one firm operating and it sells at marginal cost, so that the optimal surplus is

$$S_o = a^2/2 - F.$$

We now compare the production levels, prices, profits, and total surplus of the two game structures given by (13) and (17). Of course, as Theorem 1 assures us, $Q^s = Q^c$, $q_i^c = q_{n_s}^s$. Comparing the number of firms n_c and n_s, we see that they are equal to each other at the level of one firm ($n_c = n_s = 1$) for $a/\sqrt{F} = 2$. For smaller fixed costs, $a/\sqrt{F} > 2$, the difference in the number of active firms in the two equilibria, $n_c - n_s$, is positive and increasing in $a/\sqrt{F}$.

We now examine market concentration at the equilibria of the two games in terms of the commonly used Herfindahl–Hirshman index. The indices for the sequential and the Cournot games are

$$H_s(n_s)=(a/\sqrt{F}+1)/[3(a/\sqrt{F}-1)], \qquad H_c(n_c)=1/(a/\sqrt{F}-1),$$

respectively. Their ratio, $H_s(n_s)/H_c(n_c)=(a/\sqrt{F}+1)/3$, is larger than one, indicating the high relative concentration in the sequential game. Furthermore, this index ratio, $H_s(n_s)/H_c(n_c)$, is increasing in $a/\sqrt{F}$, the ratio of the willingness to pay over the root of the fixed cost.[14] Two factors contribute to this result. First, $H_s(n)/H_c(n)$ increases in n. The degree of inequality in quantities produced in a sequential decisions industry increases more rapidly with the number of competitors than in symmetric Cournot. Second, as the willingness to pay increases, the number of active competitors responds much more in Cournot than in sequential leadership. Thus we have a further reduction in $H_c(n_c)$ compared with $H_s(n_s)$ because $n_c > n_s$.

We also define the deadweight losses in the two models:

$$S_o-S_s=F(\log(a/\sqrt{F})/\log(2)-1/2), \qquad S_o-S_c=a\sqrt{F}-3F/2.$$

Their ratio, $(S_o-S_s)/(S_o-S_c)$, is increasing in $a/\sqrt{F}$ (decreasing in F). Thus, *the ratio of the deadweight losses varies inversely to the ratio of the Herfindahl indices,* $H_s(n_s)/H_c(n_c)$.[15]

Table 1 shows the numbers of active firms, the corresponding deviations from maximum (optimal) social surplus S_o, the ratio of the deadweight losses in Cournot and sequential leadership, and the Herfindahl indices for the Cournot and sequential leadership equilibria for varying values of the demand and cost parameters.

Note that there are significant differences in the number of active firms in the two market structures (columns 2 and 3). There are also significant differences in the deadweight losses of the two market structures. As seen in column 6 of table 1, at high levels of the demand intercept, a, or relatively

Table 1

$a/\sqrt{F}$	n_c	n_s	$(S_o-S_c)/F$	$(S_o-S_s)/F$	$(S_o-S_c)/(S_o-S_s)$	H_c	H_s	H_s/H_c
2	1	1	1/2	1/2	1	1	1	1
4	3	2	5/2	3/2	5/3	1/3	5/9	1.66
8	7	3	13/2	5/2	13/5	1/8	3/7	3.42
16	15	4	19/2	7/2	19/7	1/16	17/42	6.47
32	31	5	61/2	9/2	61/9	1/32	11/31	11.35
64	63	6	125/2	11/2	125/11	1/64	65/189	22.01

[14]For example, if the willingness to pay is fixed, the concentration index ratio $H_s(n_s)/H_c(n_c)$ is decreasing in fixed cost F.

[15]This conclusion does not change if we use other concentration indices, such as the Gini coefficient or a k-firm concentration ratio.

low fixed cost, F, *the welfare loss in Cournot compared with optimality*, $S_o - S_c$, *is a large multiple of the welfare loss of the sequential leadership game*, $S_o - S_s$, this multiple being over eleven for $a/\sqrt{F} = 64$ (last line in table 1). For the same values of the parameters, the Herfindahl index in Cournot is approximately 22 times larger than in sequential leadership!

Does the divergence between the sequential and the Cournot results continue to hold even for very small costs, i.e. as the number of active firms tends to infinity? To answer this question, we now establish limit results as fixed costs become very low. Fixing a, the intercept of the demand reduced by the marginal cost, we now let the fixed cost F go to zero, to analyze the asymptotic behavior of the two types of equilibria. Both in Cournot and the sequential models, the number of active firms tends to infinity as fixed costs go to zero, $\lim_{F\to 0} n_c = \infty$ and $\lim_{F\to 0} n_s = \infty$.

In the Cournot model, an abundance of competitors is accompanied with an almost vanishing Herfindahl index, $\lim_{F\to 0} H_c = 0$. In contrast, in sequential leadership significant inequality remains even with an infinite number of active firms, $\lim_{F\to 0} H_s = 1/3$. This is because the first firm produces half of the total production, the second one-quarter of total production, and so on.[16] Such a market is considered 'highly concentrated' in the *Merger Guidelines* (1992).

Corollary 3. As fixed costs tend to zero, the Herfindahl index of the sequential leadership game tends to 1/3, *while the Herfindahl index of the Cournot game tends to* 0.

Both models result in the efficient outcome in the limit (as $F \to 0$).[17] Thus, in both models the optimal total surplus is realized in the limit:

$$\lim_{F\to 0} S_o = \lim_{F\to 0} S_s = \lim_{F\to 0} S_c = a^2/2.$$

However, the rate of convergence is quite different. The difference between the optimal and the Cournot surplus, $S_o - S_c$, approaches zero in the order of $\sqrt{F}$, $\lim_{F\to 0}(S_o - S_c)/\sqrt{F} = a$.[18] In contrast, the difference between the optimal and the sequential surplus, $S_o - S_s$, approaches zero almost in the order of F, $\lim_{F\to 0}(S_o - S_s)/F^{1-\varepsilon} = 0$, $\forall \varepsilon > 0$.[19] Fig. 1 shows $(S_o - S_c)/\sqrt{F}$ and $(S_o - S_s)/\sqrt{F}$ against F. Thus, for any market with a finite number of firms there are significantly larger losses in surplus in the Cournot than in the

[16] Then $H_s = 1/4 + 1/16 + 1/64 + \cdots = (1/4)(1 + 1/4 + 1/4^2 + \cdots) = (1/4)(1/(1 - 1/4)) = 1/3$.

[17] Dasgupta and Ushio (1981), Fraysse and Moreaux (1981), Boyer and Moreaux (1986), and Robson (1990), have obtained similar results.

[18] $\lim_{F\to 0}(S_o - S_c)/\sqrt{F} = a - 3\sqrt{F}/2 = a$.

[19] However, $\lim_{F\to 0}(S_o - S_s)/F = \infty$.

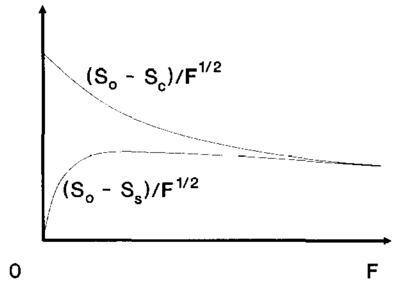

Fig. 1. Deadweight losses in SLE and Cournot equilibrium.

sequential game. The ratio of the deadweight losses tends to infinity as F goes to zero:[20]

$$\lim_{F\to 0}(S_o - S_c)/(S_o - S_s) = \infty.$$

Corollary 4. As fixed costs F tend to zero, the deadweight loss in the sequential leadership game approaches zero almost in the order of F, while the deadweight loss in the Cournot game approaches zero in the order of $\sqrt{F}$.

In summary, although surplus in both models reaches the optimal surplus in the limit, the deadweight losses are much higher in the Cournot game for any non-zero F. Furthermore, significant inequality as measured by the Herfindahl index remains in the sequential market even in the limit. Thus, even for very small fixed costs, *we observe small deadweight losses associated with high concentration in the sequential leadership model, in contrast with large deadweight losses associated with low concentration in the Cournot model.*

4. Extensions and discussion

It has been shown that sequential output decisions in a model of leaders and followers improves total surplus without causing a price increase over an equilibrium of simultaneous output (Cournot) decisions. *Strategic asymmetry and fixed costs resulted in equality in production levels, fewer active firms, and increased social welfare.* This came as a result of the more efficient utilization of the production technology in the model of sequential leadership. It was essential that firms did not have the ability to stop potential entrants from entering. This was guaranteed by a game structure in which the subgame of output decisions followed the choice of entry/exit.

[20] $\lim_{F\to 0}(S_o - S_c)/(S_o - S_s) = (a - 3\sqrt{F/2})/\{\sqrt{F}[\log(a)/\log(2) - 1/2] - \log(\sqrt{F^{\sqrt{F}}})/\log(2)\} = \infty$ since the numerator tends to a, and the denominator tends to zero. The second term of the denominator tends to zero since the quantity inside the log tends to 1, $\lim_{F\to 0}\sqrt{F^{\sqrt{F}}} = 1$.

These results for a homogeneous market can easily be extended to markets of symmetrically differentiated products where output is the strategic variable.[21] In markets where prices are the strategic variables, we cannot directly apply the methodology of this paper. In such markets it is the last firm that has the advantage, and its profits cannot be compared with the profits of a typical firm in a symmetric game. An analysis of sequential leadership with price-setting firms and free entry remains an open question.[22] Nevertheless, for markets of homogeneous goods, it is clear that in sequential leadership deadweight loss is negatively correlated with concentration. Therefore, one has to be very careful in using concentration indices [such as the Herfindahl index widely used in the *Merger Guidelines* (1992)] to judge potential deadweight losses.

Appendix

Proof of Theorem 2. Consider the output decision of firm $n_s - 2$. It is aware of its influence on the outputs of firms $n_s - 1$ and n_s. Its first-order condition is

$$\begin{aligned} &\partial \Pi_{n_s-2}(q_{n_s-2}, y)/\partial q_{n_s-2} = P(q_{n_s-2}+y) \\ &\quad + q_{n_s-2} P'(q_{n_s-2}+y)\,(1 + \mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-2} \\ &\quad + \mathrm{d}q_{n_s-1}/\mathrm{d}q_{n_s-2}(1 + \mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1})) - C'(q_{n_s-2}) = 0. \end{aligned} \tag{A.1}$$

Evaluating $\partial \Pi_{n_s-2}/\partial q_{n_s-2}$ at $q_{n_s-2} = q^s_{n_s-1}$ and using (6) we have

$$\begin{aligned} &\partial \Pi_{n_s-2}(q^s_{n_s-1}, y)/\partial q_{n_s-2} \\ &\quad = q^s_{n_s-1} P'(q^s_{n_s-1}+y)\,\mathrm{d}q_{n_s-1}/\mathrm{d}q_{n_s-2}(1 + \mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1}). \end{aligned} \tag{A.2}$$

In this expression,

$$\begin{aligned} &\mathrm{d}q_{n_s-1}/\mathrm{d}q_{n_s-2} \\ &\quad = \mathrm{d}q_{n_s-1}/\mathrm{d}y = -[\partial^2 \Pi_{n_s-1}/\partial q_{n_s-1}\,\partial y]/[\partial^2 \Pi_{n_s-1}/\partial q^2_{n_s-1}] \\ &\quad = \frac{-[P' + q^s_{n_s-1} P''(1 + \mathrm{d}q_{n_s}/\mathrm{d}y) + q^s_{n_s-1} P' \mathrm{d}^2 q_{n_s}/\mathrm{d}y^2]}{[P'(2 + \mathrm{d}q_{n_s}/\mathrm{d}y) + q^s_{n_s-1} P''(1 + \mathrm{d}q_{n_s}/\mathrm{d}y) + q_{n_s-1} P'\,\mathrm{d}^2 q_{n_s}/\mathrm{d}y^2 - C'']}. \end{aligned} \tag{A.3}$$

[21]These results do not directly extend to locationally differentiated markets because in such markets each firm's product is not equally substitutable for any other firm's product.

[22]In discussing sequential location in a variant of Hotelling's (1929) model, Economides and Howell (1991) show that the order of profits does not always follow the order of action.

Since $\mathrm{d}q_{n_s}/\mathrm{d}y<0$ and $\mathrm{d}^2q_{n_s}/\mathrm{d}y^2>0$, both terms in brackets in the numerator and the denominator of (A.3) are negative, so that $\mathrm{d}q_{n_s-1}/\mathrm{d}q_{n_s-2}<0$. The last term of (A.2) is positive:

$$(1+\mathrm{d}q_{n_s}/\mathrm{d}q_{n_s-1})=(P'-C'')/(2P'+q_{n_s-1}P''-C'')>0,$$

because both numerator and denominator are negative. Therefore $\partial\Pi_{n_s-2}(q^{\mathrm{s}}_{n_s-1},y)/\partial q_{n_s-2}>0$. Since Π_{n_s-2} is concave, it follows that $q^{\mathrm{s}}_{n_s-2}>q^{\mathrm{s}}_{n_s-1}$. Q.E.D.

Proof of Theorem 5. The last (nth) firm maximizes

$$\Pi_n=q_n\left(a-\sum_{i=1}^{n-1}q_i-q_n\right)-F$$

at

$$q^{\mathrm{m}}_n=\left(a-\sum_{i=1}^{n-1}q_i\right)\Big/2. \tag{A.4}$$

The $n-1$st firm maximizes

$$\Pi_{n-1}=q_{n-1}\left(a-\sum_{i=1}^{n-2}q_i-q_{n-1}-q^{\mathrm{m}}_n\right)-F$$

$$=q_{n-1}\left(a-\sum_{i=1}^{n-2}q_i-q_{n-1}\right)\Big/2-F$$

after substituting from (A.4). Π_{n-1} is maximized at

$$q^{\mathrm{m}}_{n-1}=\left(a-\sum_{i=1}^{n-2}q_i\right)\Big/2.$$

In general, the $n-j$th firm maximizes

$$\Pi_{n-j}=q_{n-j}\left(a-\sum_{i=1}^{n-j-1}q_i-q_{n-j}-\sum_{k=1}^{j}q^{\mathrm{m}}_{n-j+k}\right)-F. \tag{A.5}$$

Firm $n-j$ recognizes the influence of its quantity choice q_{n-j} on quantities chosen later. On these latter ones we have put the superscript m. The claim below establishes the exact value of the total output for firms that act after firm $n-j$.

Claim. For all $j=1,\dots,n-1$,

$$\sum_{k=1}^{j} q^{\mathrm{m}}_{n-j+k}=\left(a-\sum_{i=1}^{n-j} q_i\right)(1-2^{-j}). \tag{A.6}$$

Proof. The proof of the claim is by mathematical induction. As eq. (A.4) shows, (A.6) holds for $j=1$. We show that if (A.6) holds for $j=t$, then it also holds for $j=t+1$. Then the result follows by mathematical induction.

Assuming that (A.6) holds for $j=t$, substituting (A.6) in (A.5) yields

$$\Pi_{n-t}=q_{n-t}\left(a-\sum_{i=1}^{n-t-1} q_i-q_{n-t}\right)\Big/2^t. \tag{A.7}$$

Firm $n-t$ maximizes Π^{n-t} at

$$q^{\mathrm{m}}_{n-t}=\left(a-\sum_{i=1}^{n-t-1} q_i\right)\Big/2. \tag{A.8}$$

Therefore

$$\sum_{k=1}^{t+1} q^{\mathrm{m}}_{n-t-1+k}=\sum_{k=0}^{t} q^{\mathrm{m}}_{n-t+k}=\sum_{k=1}^{t} q^{\mathrm{m}}_{n-t+k}+q^{\mathrm{m}}_{n-t}.$$

Substituting the first term from (A.6) and the second from (A.8) yields

$$\sum_{k=1}^{t+1} q^{\mathrm{m}}_{n-t-1+k}=(1-2^{-j})\left(a-\sum_{i=1}^{n-t-1} q_i\right)(1-1/2)+\left(a-\sum_{i=1}^{n-t-1} q_i\right)\Big/2$$

$$=\left(a-\sum_{i=1}^{n-t-1} q_i\right)(1-2^{-t-i}),$$

which is eq. (A.6) for $j=t+1$. Thus, the claim is proved. Q.E.D.

Therefore, it is clear that the profit function of firm $n-t$ takes the form of (A.7), and is maximized at q^{m}_{n-t} given by (A.8). For $t=n-1$, we have from (A.8) that $q^{\mathrm{m}}_1=a/2$. It follows that $q^{\mathrm{m}}_2=a/4$, $q^{\mathrm{m}}_3=a/8$, and in general

$$q^{\mathrm{m}}_i=a/2^i, \tag{A.9}$$

i.e. outputs form a geometric sequence. Q.E.D.

References

Basu, K. and N. Singh, 1990, Entry deterrence in Stackelberg-perfect equilibria, International Economic Review 31, no. 1, 61–71.

Boyer, M. and M. Moreaux, 1986, Perfect competition as the limit of a hierarchical market structure, Economics Letters 22, 115–118.

Cournot, A., 1960, Researchers into the mathematical principles of the theory of wealth (Kelly, New York). (Original published in French in (1838), English translation by N.T. Bacon.)

Dasgupta, P. and Y. Ushio, 1981, On the rate of convergence of oligopoly equilibria in large markets: An example, Economics Letters 8, 13–17.

Daughety, A., 1990, Beneficial concentration, American Economic Review 80, no. 5, 1231–1237.

Department of Justice, 1992, Merger guidelines (Washington, DC).

Dixit, A., 1979, A model of duopoly suggesting a theory of entry barriers, Bell Journal of Economics 10, 20–32.

Eaton, B. C. and R. Ware, 1987, A theory of market structure with sequential entry, Rand Journal of Economics 18, no. 1, 1–16.

Economides, N., 1989, Symmetric equilibrium existence and optimality in differentiated products markets, Journal of Economic Theory 47, no. 1, 178–194.

Economides, N. and J. Howell, 1991, Does it pay to be first? Sequential choice of locations of differentiated products, EC-91-24, Stern School of Business, New York University.

Fraysse, J. and M. Moreaux, 1981, Cournot equilibrium in large markets under increasing returns, Economics Letters 8, 217–220.

Gilbert, R., 1986, Pre-emptive competition, in: G.F. Mathewson and J. Stiglitz, eds., New developments in the analysis of market structure (MIT Press, Cambridge, MA).

Gilbert, R. and X. Vives, 1986, Entry deterrence and the free rider problem, Review of Economic Studies 53, no. 1, 71–83.

Hotelling, H., 1929, Stability in competition, Economic Journal 39, no. 2, 137–175.

Kreps, D. and J. Scheinkman, 1983, Quantity precommitment and Bertrand competition yield Cournot outcomes, Bell Journal of Economics 14, 326–337.

Novshek, W., 1984, Finding all n-firm equilibria, International Economic Review 25, no. 1, 61–70.

Nti, K. and M. Shubik, 1981, Non-cooperative oligopoly with entry, Journal of Economic Theory 24, 187–204.

Robson, A., 1990, Duopoly with endogenous strategic timing: Stackelberg regained, International Economic Review 31, 263–274.

Robson, A., 1990, Stackelberg and Marshall, American Economic Review 80, no. 1, 69–82.

Saloner, G., 1987, Cournot duopoly with two production periods, Journal of Economic Theory 42, 183–187.

Salop, S., 1979, Monopolistic competition with outside goods, Bell Journal of Economics 10, 141–156.

Seade, J., 1980, On the effects of entry, Econometrica 48, no. 2, 479–490.

von Stackelberg, H., 1934, Marktform und Gleichgewicht (Julius Springer, Vienna).

Vives, X., 1988, Sequential entry, industry structure and welfare, European Economic Review 32, 1671–1687.

Zink, H., 1990, Increasing returns, quality uncertainty, product differentiation, and counter-cyclical price mark-ups, Mimeo.