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STABLE CARTELS*

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1. INTRODUCTION

It is often argued that cartel arrangements are inherently unstable: under the assumption that their actions will not be imitated, individual members of the cartel have an incentive to increase output beyond the point of joint profit maximization; at the latter, marginal revenue exceeds marginal cost. This is well taken and, for homogeneous cost and demand conditions, firms in the competitive fringe do indeed enjoy a higher level of profits than firms in the cartel. The decision of a firm to break away from the cartel and join the competitive fringe does however affect the market structure, at least in a finite economy. It is then possible that the fall in price due to the increased overall competition in the market leads to a fall in profit for the individual firm contemplating a move out of the cartel which goes beyond the advantage of joining the competitive fringe.

We consider a market or industry in which a subset of firms form a cartel which acts as a price leader, and we give a definition of cartel stability which allows firms to recognize the impact of their actions on the overall market structure. To pose the problem and prove the existence of a stable cartel was the original contribution of d'Aspremont, Jacquemin, Jaskold-Gaszewicz and Weymark [1983].[2] We give an alternative proof of existence for the case of linear demand and marginal cost functions, which allows us to study the characteristics of stable cartels, such as uniqueness and size.

We show that the stable cartel is unique as long as firms are not too cost-efficient relative to market demand. Otherwise, there exist industry sizes for which two cartels are stable, one of which comprises all firms in the industry. Furthermore, we show that the relative size of stable cartels is a decreasing function of the

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[2] The stability of collusive arrangements is closely related to the question of incentives for price taking, competitive behavior. Drèze and Gabszewicz [1971] have derived a negative result for the stability of collusive arrangements ("syndicates") in large, atomless economies. Johansen [1977], commenting on Postlewaite and Roberts [1977], has raised the possibility of collusive behavior as an objection to competitive assumptions. Our argument demonstrates that his point is well taken except for the limiting case of an infinite economy. The reply of Postlewaite and Roberts [1976] refers, of course, precisely to this case.

size of the economy; as intuition suggests, it tends to zero as the size of the market tends to infinity relative to the size of individual firms.

The paper is organized as follows: in Section 2, we determine the equilibrium price, and levels of output and profit for firms inside as well as outside the cartel for a given cartel size. In Section 3, we characterize stable cartels. In Section 4, we conclude.

2. EQUILIBRIUM

A single, homogeneous good is produced by N firms indexed by a subscript, i, $i \in \{1, \ldots, N\}$. Firm i produces quantity q_i incurring cost $c_i = (1/2c)q_i^2$, $c > 0$; that is, firms face identical cost conditions and marginal cost is linear and increasing in output. Given total output $Q = \sum_{i=1}^{N} q_i$, the price is determined by the inverse demand function $p = (1/bN)(aN - Q)$, $a > 0$, $b > 0$, $0 \leqq Q \leqq aN$. That the number of firms (production units) in the market be proportional to ouput demanded is natural in the study of the effects of size in the presence of increasing marginal cost. The structure prevailing in the market is described by the pair (N, α), where $\alpha N = k \in \{1, \ldots, N\}$. The interpretation is the following: Firms in $L = \{1, \ldots, k\}$, the leaders, constitute a cartel and set the price, p; firms in $F = \{k+1, \ldots, N\}$, the competitive fringe, behave atomistically and choose each $q_{f,i}(p)$ so as to maximize profit, treating parametrically the price level. That is $q_{f,i}(p) = pc$, and for the fringe as a whole

$$Q_f(p) = \sum_{i=k+1}^{N} q_{f,i}(p) = (1-\alpha)Nq_f(p) = (1-\alpha)Npc. \tag{1}$$

The parameter α thus measures the size of the cartel relative to the size of the economy, N. The residual demand function[3] facing the cartel is given by $Q_l(p) = Q(p) - Q_f(p) = N(a - bp) - Q_f(p)$. The cartel has full knowledge of the impact of a change in price on the output of the competitive fringe. Each firm in the cartel produces q_l; since there are αN firms in the cartel, $q_l = (1/\alpha N)Q_l$. Total cost for the cartel is then $C(Q_l) = \alpha N(1/2c)(Q_l/\alpha N)^2 = Q_l^2/2c\alpha N$. The price which maximizes individual as well as total profits for the cartel is given by

$$p(Q_f) = \frac{\left(a - \frac{Q_f}{N}\right)\phi(\alpha)}{(\alpha c + b)\phi(\alpha) - \alpha^2 c^2}, \tag{2}$$

where $\phi(\alpha) = b + c\alpha$. The equilibrium price is obtained by substituting (1) into (2) and solving

$$p(\alpha) = \frac{a(b+c)}{(b+c)^2 - \alpha^2 c^2}. \tag{3.1}$$

[3] The appropriate definition of residual demand is not a priori obvious; it depends on the underlying model. The definition we adopt here is a standard one in the literature; alternative specifications may, however, affect the results in a non-trivial manner.

Due to the specification of the demand function, the size of the economy, N, does not affect the equilibrium. As a consequence, we omit it as an argument in the equilibirum price, quantity, and profit functions. The levels of ouput and profit at equilibrium for firms inside as well as outside the cartel follow directly from (3.1):

$$q_l = \frac{ac}{b+c+\alpha c}\,; \tag{3.2}$$

$$\pi_l(\alpha) = \frac{a^2c}{2[(b+c)^2-\alpha^2c^2]}\,; \tag{3.3}$$

$$q_f(\alpha) = \frac{ac(b+c)}{(b+c)^2-\alpha^2c^2}\,; \tag{3.4}$$

$$\pi_f(\alpha) = \frac{a^2c(b+c)^2}{2[(b+c)^2-\alpha^2c^2]^2}\,. \tag{3.5}$$

To facilitate comparison we present the outcomes corresponding to price taking behavior and joint maximization indexed by superscripts W and M respectively:

$$p^W = \frac{a}{b+c}\,; \tag{4.1}$$

$$q^W = \frac{ac}{b+c}\,; \tag{4.2}$$

$$\pi^W = \frac{a^2c}{2(b+c)^2}\,. \tag{4.3}$$

$$p^M = \frac{a(b+c)}{(b+c)^2-c^2}\,; \tag{5.1}$$

$$q^M = \frac{abc}{(b+c)^2-c^2}\,; \tag{5.2}$$

$$\pi^M = \frac{a^2c}{2[(b+c)^2-c^2]}\,. \tag{5.3}$$

3. STABILITY

The stability of a cartel depends on the profits that are generated at equilibrium for firms inside and outside the cartel: A cartel is stable if and only if firms inside do not find it desirable to exit and firms outside do not find it desirable to enter. In deciding the desirability of a move, a firm, we postulate, hypothesizes that no other firm will change its strategy concerning membership in the cartel. Furthermore firms are assumed to have exact knowledge of the dependence of profits at equilibirum on the size of the cartel; i.e. they know the functions $\pi_l(\alpha)$ and $\pi_f(\alpha)$ derived from (3.3) and (3.5) respectively. This latter knowledge is indispensable, since a move by a firms affects the size of the cartel and hence the resulting level and distribution of profits.[4]

A market structure of cartel (α, N) is internally stable if and only if

$$\pi_f\left(\alpha - \frac{1}{N}\right) \leqq \pi_l(\alpha);$$

it is externally stable if and only if

$$\pi_l\left(\alpha + \frac{1}{N}\right) \leqq \pi_f(\alpha);$$

it is stable if and only if it is both externally and internally stable.[5] Internal stability guarantees that no member of the cartel desires to exit, while external stability guarantees that no firm in the fringe desires to enter the cartel. External stability at $\alpha = 1$ and internal stability at $\alpha = 0$ hold *a fortiori.*

PROPOSITION 1. *A stable cartel always exists. If firms are not too cost efficient relative to market demand, i.e. if* $(b/c) > (k/(k^2-1)^{1/2}) - 1$, *where* $k = 8/(1+\sqrt{17})$, *the stable cartel is unique;*[6] *otherwise there exist two critical sizes for the economy,* $\overline{\overline{N}} > \overline{N} > 0$, *such that: for* $\overline{\overline{N}} > N > \overline{N}$, *two stable cartels exist, one of which comprises all firms in the industry; for* $N > \overline{\overline{N}}$ *(resp.* $N < \overline{N}$*) the stable cartel is unique and does not (resp. does) comprise all firms in the industry.*

The proof of Proposition 1 is based on two lemmas which are interesting in their own right.

LEMMA 1.

(i) $\dfrac{\partial p(\alpha)}{\partial \alpha} > 0, \dfrac{\partial^2 p(\alpha)}{\partial \alpha^2} > 0, p(0) = p^W, p(1) = p^M;$

[4] The issue of cartel stability raised here is of course a special case of the stability of coalition structures in a general game theoretic framework. Even though an explicit specification of a coalition structure does not enter into the definition of some of the standard solution concepts (such as the Shapley value, the core, or the von Neumann-Morgenstern solution), these concepts can easily be defined with respect to a given coalition structure — Aumann and Drèze [1974]; and once a solution concept with respect to a given coalition structure has been defined, the question of stability of a given coalition structure can be raised. This route is followed in the framework of an abstract game by Shenoy [1979] and, more pertinently, by Hart and Kurz [1983] for the case of the coalition structure (Shapley) value. Even though our solution concept for a given coalition (cartel) structure is derived from the specific market structure we consider and does not coincide with the corresponding value, our definition of cartel (coalition structure) stability is very much along the lines of Hart and Kurz. In particular, agents contemplating a move out of a coalition do indeed perceive the impact on the payoff they can anticipate resulting from the change in the coalition structure; furthermore, the defection of one of its members does not necessarily lead to the total dissolution of a coalition. Economides [1985] has extended our approach to an abstract framework.

[5] It may seem that lack of external (as opposed to internal) stability poses no threat to a cartel. This is not clear, however, especially when the profits of the cartel are an increasing function of its size as is most often the case.

[6] This and subsequent statements concerning the number of stable cartels are strictly speaking true only "barring pathology" in a sense which will be clear in the proofs.

(ii) $\pi_l(0) = \pi_f(0) = \pi^W$; $\pi_l(1) = \pi^M$;

(iii) $\pi_f(\alpha) > \pi_l(\alpha)$ *for all* α *in* $(0, 1]$;

(iv) $\dfrac{\partial \pi_f(\alpha)}{\partial \alpha} > \dfrac{\partial \pi_l(\alpha)}{\partial \alpha} > 0$ for all α in $(0, 1)$;

(v) $\dfrac{\partial^2 \pi_f(\alpha)}{\partial \alpha^2} > \dfrac{\partial^2 \pi_l(\alpha)}{\partial \alpha^2} > 0$ for all α in $(0, 1)$.

LEMMA 2. *The inverse functions $\alpha_l(\pi)$ and $\alpha_f(\pi)$ of $\pi_l(\alpha)$ and $\pi_f(\alpha)$, respectively, are well defined on the interval $[\pi^W, \pi^M]$. The difference $\alpha_l(\pi)-\alpha_f(\pi)$ (i.e. the horizontal distance between $\pi_l(\alpha)$ and $\pi_f(\alpha)$, increases monotonically with π for high cost markets $\left(\text{i.e. for } \frac{b}{c} > \left(k/(k^2-1)^{1/2}\right)-1,\ k=\frac{8}{1+\sqrt{17}}\right)$ for low cost markets $\left(\text{i.e. for } \frac{b}{c} < \left(k/(k^2-1)^{1/2}\right)-1,\ k=\frac{8}{1+\sqrt{17}}\right)$ the difference $\alpha_l(\pi)-\alpha_f(\pi)$ increases for π in $[\pi^W, \bar{\pi})$ and decreases for π in $(\bar{\pi}, \pi^M]$, attaining a maximum at $\bar{\pi}=\pi^W k^2$.*

The proofs of Lemmas 1 and 2 are given in the Appendix. The intuition behind the argument for Proposition 1 can be explained by considering the simple case where the horizontal distance between the profit functions $\pi_l(\alpha)$ *and* $\pi_f(\alpha)$ starts from 0 at $\alpha=0$ and increases monotonically with α attaining a maximum at $\alpha=1$. If this distance is less than or equal to $(1/N)$ at $\alpha=1$, then $\alpha^*=1$ is a stable cartel; furthermore it is unique since, by monotonicity, external stability fails for all α in $[0, 1)$. If the horizontal distance is larger than $(1/N)$ at $\alpha=1$, then there exists a unique π^* in (π^W, π^M) such that $\alpha_l(\pi^*)-\alpha_f(\pi^*)=1/N$. Barring pathology, there is a unique α^* in $[\alpha_l(\pi^*), \alpha_f(\pi^*)]$ such that $\alpha^* N=k^* \in \{1,\ldots,(N-1)\}$. It is clear that α^* is a stable cartel; it is furthermore unique since, by monotonicity, external stability fails for α less than α^* and internal stability fails for α larger than α^*. If monotonicity of the horizontal distance fails, as it does indeed for low cost markets, the uniqueness of the stable cartel may fail as well. It should be pointed out, however, that the stable cartel sizes are at most two and that, furthermore, in the case of multiple stable cartels one of them always comprises all firms in the industry.

PROOF OF PROPOSITION 1. Let π^* be the profit level(s) in the closed interval $[\pi^W, \pi^M]$ at which the horizontal distance between the profit functions is equal to $(1/N)$

$$\alpha_l(\pi^*) - \alpha_f(\pi^*) = \frac{1}{N}. \tag{6}$$

From Lemmas 1 and 2 it follows that $\pi^* > \pi^W$ and that three cases may occur: there may be zero, one or two values π^* which satisfy (6).

Consider first the case at which (6) holds for no π^* in $[\pi^W, \pi^M]$. Since $\alpha_l(\pi^W) = \alpha_f(\pi^W)$, it follows that $[\alpha_l(\pi^M)-\alpha_f(\pi^M)] < (1/N)$ and $\alpha^*=1$ is a stable cartel. Furthermore, external uniqueness fails everywhere and hence $\alpha^*=1$ is the unique stable cartel.

We next consider the case at which the solution to (6) is unique. It follows from Lemma 2 that from π^W to π^* the horizontal distance $[\alpha_l(\pi)-\alpha_f(\pi)]$ increases monotonically starting from 0 and stays less than $(1/N)$ which it attains at π^*; furthermore, as π varies from π^* to π^M it is always greater than or equal to $(1/N)$, but it need not increase monotonically — it may increase and then decrease. Barring pathology,[7] there exists a unique element α^* in $\left\{\frac{1}{N},\ldots,1-\frac{1}{N}\right\}$ in the closed interval $[\alpha_f(\pi^*),\ \alpha_l(\pi^*)]$. It can be readily shown that α^* is a stable cartel. Let π^e be the profits attained by the fringe at α^*; since the horizontal distance between the profit functions exceeds $(1/N)$ for all π in $(\pi^*,\ \pi^M)$, $\pi^e>\pi_l(\alpha^*+(1/N))$ and external stability follows. Internal stability is established by an analogous argument. Uniqueness follows trivially: For all values of $\alpha>\alpha^*$ (resp. $\alpha<\alpha^*$) internal stability (resp. external stability) fails, since the horizontal distance between the profit functions is greater than (resp. less than) $1/N$ for $\alpha>\alpha^*$ (resp. $\alpha<\alpha^*$).

Consider finally the case in which (6) is satisfied by two values of π. It follows from Lemma 2 that one solution, π_1^*, lies in the open interval $(\pi^W,\ \bar{\pi})$ and the other, π_2^*, in the interval $(\bar{\pi},\ \pi^M]$. Concerning π_1^*, the situation is effectively identical to the one considered immediately previously and the unique (again, barring pathology) element α_1^* of $\left\{\frac{1}{N},\ldots,1-\frac{1}{N}\right\}$ in the closed interval $[\alpha_f(\pi_1^*),\ \alpha_l(\pi_1^*)]$ constitutes a stable cartel. Concerning π_2^*, observe that no element α_2^* of $[\alpha_f(\pi_2^*),\ \alpha_l(\pi_2^*)]$ can be stable. External stability fails since the horizontal distance is less than $(1/N)$ as π varies from π_2^* to π^M. On the other hand, $\alpha^*=1$ is itself a stable cartel since it is internally stable.[8,9]

To complete the argument note that for N large $[\alpha_l(\pi^M)-\alpha_f(\pi^M)]>(1/N)$; this gives $\bar{N}$. The value of $\bar{N}$ is obtained as the solution to the equation $[\alpha_l(\bar{\pi})-\alpha_f(\bar{\pi})]=(1/\bar{N})$, where, from Lemma 2, $\bar{\pi}$ is the value of profits at which the horizontal distance between the profit function switches from being an increasing to being a decreasing function of the size of the cartel. Q.E.D.

Figures 1 and 2 illustrate Proposition 1.

REMARK 1. An alternative argument developed by d'Aspremont et al. [1983] shows that the existence of stable cartels is independent of the shape of the profit functions. The argument is as follows: Consider the functions $\pi_f(\alpha)$ and $\pi_l(\alpha)$ defined on $[0,\ 1]$. At $\alpha=0$ internal stability holds a fortiori; if external stability holds as well, $\alpha^*=0$ is a stable cartel. If external stability fails at $\alpha=0$, i.e. $\pi_l(1/N)>\pi_f(0)$, either $\pi_l(k+1)/N)>\pi_f(k/N)$ for all $k\in\{0,\ldots,N-1\}$ or there

7 Two pathologies may occur: $\alpha_f(\pi^*)$, and hence $\alpha_l(\pi^*)$ as well, may be multiples of $(1/N)$; in this case there are two stable cartels with one more firm in one than in the other. Or π^* may be equal to π^M, in which case $\alpha^*=1$ and $\alpha^*=1-(1/N)$ are both stable.

8 Note that if $\bar{\pi}=\pi_1^*=\pi_2^*$ there is a unique stable cartel of size $\alpha^*=1$.

9 Note that if $\pi_2^*=\pi^M$ there are two stable cartels in $(\bar{\pi},\ \pi^M]$: $\alpha^*=1$ and $\alpha^*=1-1/N$; this is again the pathology we have been excluding all along.

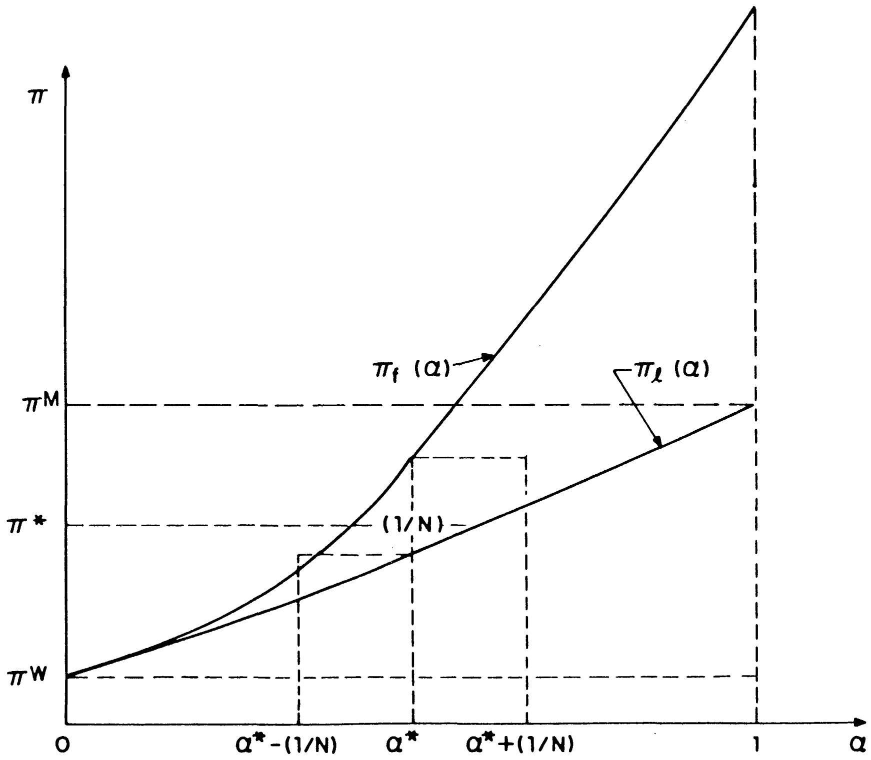

FIGURE 1

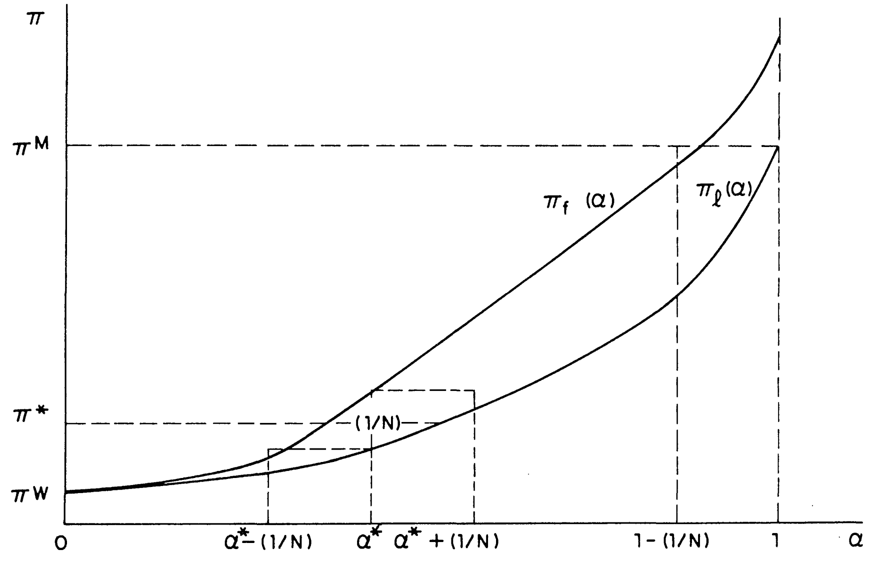

FIGURE 2

exists $k^* \in \{0,\ldots, N-1\}$ such that $\pi_f((k^*+1)/N) \leqq \pi_f(k^*X/N)$ while $\pi_f\{(k+1)/N\} > \pi_f(k/N)$ for all $k \in \{0,\ldots, k^*-1\}$. In the first case $\alpha^* = 1$ while in the second case $\alpha^* = k^*/N$ are stable cartels. This general argument does not however allow for the characterization, in particular the uniqueness, of stable cartels.

The following two corollaries characterize the dependence of the size of stable cartels on the size of the economy on the one hand and on relative cost and demand conditions on the other. We restrict our attention to stable cartels which do not comprise all firms in the industry.

COROLLARY 1. *The relative size of the stable cartel which does not comprise all firms in the industry is a decreasing function of the size of the economy. In the infinite economy, the only stable relative size of a cartel is zero.*

COROLLARY 2. *The relative size of the stable cartel which does not comprise all firms in the industry is increasing in cost efficiency, c, consumers' willingness to pay, a, and is decreasing in the slope of the demand function, b.*

The proofs of Corollaries 1 and 2 are given in the Appendix.

Figure 3 depicts the variation of the size of the stable cartel(s) with the size of the economy, N.[10]

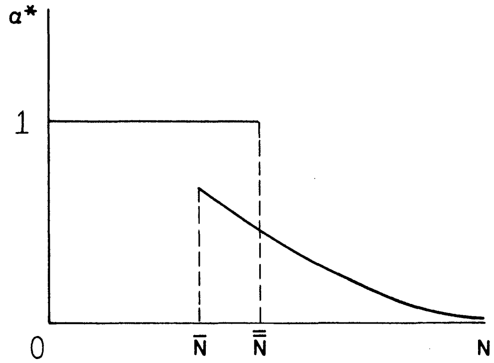

FIGURE 3

[10] The concept of cartel stability we have considered is static; that is, no mention is made of the tendency of the cartel to tend to its stable size. The adjustment in the size of the cartel away from a (statically) stable point α^* can be naturally specified as follows: if at $\alpha \neq \alpha^*$ external stability fails while internal stability holds the cartel size increases; if external stability holds while internal stability fails the outcome is ambiguous. Consequently, we may define a stable cartel size α^* to be dynamically stable if and only if, in a (large) open neighborhood of α^*, external stability holds while internal stability fails for $\alpha > \alpha^*$ and vice versa for $\alpha < \alpha^*$. Then it is easily shown that under pure price leadership, stable cartels are dynamically stable in large open neighborhoods.

4. CONCLUSION

We have provided an argument for the stability of cartel arrangements. Even Even though it is always preferable for a firm to be outside rather than inside the cartel, when each firm recognizes the effect of its movement into or out of the cartel on the equilibrium price, a stable cartel exists. Furthermore, when firms are not too cost efficient relative to market demand, the stable cartel is unique. Otherwise, there may exist two stable cartels, one of which comprises all firms in the industry.[11]

The extension of the argument to a less narrow, preferably general equilibrium, dynamic framework is an open question.[12]

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APPENDIX

PROOF OF LEMMA 1

(i) follows by direct computation.

Set $\alpha=0$ (resp. $\alpha=1$) in equation (3.3) and (4.3) (resp. (5.3)) obtains. Similarly, set $\alpha=0$ in equation (3.5) and (4.3) obtains. Thus (ii) follows.

Divide equation (3.5) by equation (3.3); after simplification the following obtains:

$$\frac{\pi_f(\alpha)}{\pi_l(\alpha)} = \frac{(b+c)^2}{(b+c)^2-\alpha^2c^2} > 1 \text{ for } \alpha \text{ in } (0,\ 1);$$

therefore (iii) follows.

[11] Donsimoni [1985] has extended the argument to allow for heterogeneous firms.

[12] Whether prices or quantities are the strategic variable employed by firms cannot be determined on a priori grounds. Furthermore, depending on the strategic variable adopted, different equilibrium outcomes are likely to obtain. We may look briefly into the case in which the cartel sets its level of output as opposed to the price. Intuitively, other things being equal, quantity as opposed to price setting by the cartel leads to a higher price: The competitive fringe does not perceive itself deprived of all price making power and hence reduces output below the level where price equals marginal cost. The equilibrium price and distribution of profits depend on the prevailing market structure (N, α), where N indicates the size of the economy, and α the relative size of the cartel. Observe that the market power perceived by the competitive fringe in the present context prevents its response, and hence the equilibrium configuration, from being independent of the size of the economy, N. The existence of a stable cartel poses no problem. It suffices to recall Remark 1: the argument for the existence of stable cartels is independent of the shape of the profit functions. Thus a stable cartel exists for any finite market under quantity strategies. The characterization and in particular the uniqueness of stable cartels is not straightforward.

Differentiate equations (3.3) and (3.5) with respect to α; (iv) follows:

$$\text{(A–1)} \qquad \frac{\partial \pi_f(\alpha)}{\partial \alpha} = \frac{a^2c(b+c)^2}{2}\left(\frac{4\alpha c^2}{[(b+c)^2-\alpha^2c^2]^3}\right) > 0$$

$$\frac{\partial \pi_l(\alpha)}{\partial \alpha} = \frac{a^2c}{2}\left(\frac{2\alpha c^2}{[(b+c)^2-\alpha^2c^2]^2}\right) > 0.$$

Differentiate equation (A–1) with respect to α and (v) follows. Q. E. D.

PROOF OF LEMMA 2

From Lemma 1, it follows that $\pi_l(\alpha)$ and $\pi_f(\alpha)$ have well defined inverses. From equations (3.3) and (3.5), these inverses can be written:

$$\text{(A–2–1)} \qquad \alpha_f(\pi) = (1+\gamma)\left(1-\left(\frac{\pi^W}{\pi}\right)^{1/2}\right)^{1/2}$$

$$\text{(A–2–2)} \qquad \alpha_l(\pi) = (1+\gamma)\left(1-\frac{\pi^W}{\pi}\right)^{1/2},$$

where $\gamma = b/c$, for π in (π^W, π^M).

Differentiate equation (A–2) with respect to α, divide by $(1+\gamma)\dfrac{\pi^W}{2\pi^2}$:

$$\text{(A–3)} \quad \frac{\partial \alpha_l(\pi)}{\partial \pi} - \frac{\partial \alpha_f(\pi)}{\partial \pi} = \left(1-\frac{\pi^W}{\pi}\right)^{-1/2} - \frac{1}{2}\left(\left(\frac{\pi^W}{\pi}\right)^{-1/2}\left(1-\left(\frac{\pi^W}{\pi}\right)^{1/2}\right)^{-1/2}\right).$$

The sign of equation (A–3) is that of a polynomial of degree two in $\left(\dfrac{\pi^W}{\pi}\right)^{1/2}$ with one positive root: $\left(\dfrac{\pi^W}{\pi}\right)^{1/2} = \dfrac{1}{k} = \dfrac{1+\sqrt{17}}{8}$. Thus for $\pi \gtreqless k^2\pi^W$, expression (A–3) is $\gtreqless 0$. Furthermore $\pi^M = \dfrac{(1+\gamma)^2}{(1+\gamma)^2-1}\pi^W$. So, for $\gamma < \bar{\gamma} = (k/(k^2-1)^{1/2}) - 1$, $\bar{\pi} = k^2\pi^W$ is strictly bigger than π^M and the horizontal distance between the profit functions is monotonically increasing. For $\gamma > \bar{\gamma}$ two situations prevail: the horizontal distance increases monotonically as π varies from π^W to $\bar{\pi}$, while it monotonically decreases as π varies from $\bar{\pi}$ to π^M. Q. E. D.

PROOF OF COROLLARY 1

Equation (6) which defines the equilibrium condition can be rewritten as:

$$F(\pi^*, \omega) = \alpha_l(\pi^*) - \alpha_f(\pi^*) - \frac{1}{N} = 0.$$

To examine the impact of a change in N on α^*, it suffices to examine its impact on π^* since α^* and π^* are positively related.

Thus differentiate $F(\pi^*, \omega)$ with respect to π^*

$$\frac{\partial F(\pi^*, \omega)}{\partial \pi^*} = \frac{\partial \alpha_l(\pi^*)}{\partial \pi^*} - \frac{\partial \alpha_f(\pi^*)}{\partial \pi^*},$$

which is positive for all equilibrium values of π^* different from π^M. Corollary (1) follows then immediately:

$$\frac{dN}{d\pi^*} = -\frac{\partial F(\pi, \omega)}{\partial \pi^*} \bigg/ \frac{\partial F(\pi^*, \omega)}{\partial N} < 0. \qquad \text{Q. E. D.}$$

PROOF OF COROLLARY 2

To establish the impact of a change of a, b, and c on π^*, it suffices to compute the partial derivative of $F(\pi^*, \omega)$ with respect to these parameters. Taking the partial derivative of F the following obtains:

$$\frac{d\pi^*}{da} = \frac{\partial F(\pi^*, \omega)}{\partial a} \bigg/ \frac{\partial F(\pi^*, \omega)}{\partial \pi^*} > 0$$

$$\frac{d\pi^*}{db} = \frac{\partial F(\pi^*, \omega)}{\partial b} \bigg/ \frac{\partial F(\pi^*, \omega)}{\partial \pi^*} < 0$$

$$\frac{d\pi^*}{dc} = \frac{\partial F(\pi^*, \omega)}{\partial c} \bigg/ \frac{\partial F(\pi^*, \omega)}{\partial \pi^*} > 0 \qquad \text{Q. E. D.}$$

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