The Division of Markets Is Limited by the Extent of Liquidity (Spatial Competition with Externalities)

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Liquidity considerations will limit the number of markets in a competitive economy. Welfare implications are ambiguous. Since liquidity is a positive externality, there may be too little liquidity per market at a noncooperative equilibrium and too many markets compared to the surplus-maximizing market structure. But liquidity is also self-reinforcing. Given an existing equilibrium, new markets may not open because nobody wants to use a new market with low liquidity. There may be too few markets to achieve efficiency and new markets will not open. A nondiscriminating monopolist will operate smaller and more numerous markets compared to optimality as well as to the equilibrium of independent auctioneers.

How many markets are there in a competitive economy? In the standard general equilibrium model (for example, Gerard Debreu, 1959), where there is no trading friction, there are as many markets as there are commodities. When there are trading frictions due to technological or demographic constraints, the number of markets may be less than the number of commodities (for example, David Cass and Karl Shell, 1983; Peter Diamond, 1982; Robert Townsend, 1983).

This paper studies the role of lack of liquidity as an endogenous trading friction in limiting the number of markets in a competitive economy. In many markets, the variance of (competitive) price fluctuations is negatively correlated with the volume of trade in that market. We define a market as having *high liquidity* when the volume of trade is high and the corresponding variance of price is low.[1] The problem of liquidity is most apparent in financial markets. For example, liquidity is a particularly important factor in determining the success of futures contracts. The futures market for any asset has only a small number of maturity dates. In principle, many more maturity dates may be admitted. However, if there were many maturity dates, the market at each maturity date would be thin. Market participants may face large competitive price fluctuations arising only from the thinness of the markets. Traders may prefer fewer maturity dates so that liquidity is enhanced in the remaining markets, even though they will have fewer maturity dates to choose from. Thus, there is a fundamental tradeoff between liquidity and the number of markets.[2] In a geographic context, the historical development of spa-

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[1] Our definition ignores the speed at which sales can be consummated. For example, Steven Lippman and John McCall (1986) define an asset as liquid "if it can be sold quickly and at a predictable price." Their paper contains a comprehensive discussion of the attributes of liquidity that are important to a seller of an asset.

[2] See Dennis Carlton, 1984; Kenneth Garbade and William Silber, 1979; Lester Telser, 1981, and the references therein for discussions of liquidity and the success of futures markets. Deborah Black (1985) contains empirical evidence on the same subject.

tially separated towns may be attributed to the tradeoff that farmers faced between liquidity in the trading place and costs of transporting the commodities to the market.

This paper was motivated by Diamond's search models (for example, Peter Diamond, 1982, 1984; Dale Mortensen, 1976; Aloysius Siow, 1982). Many issues addressed in this paper were raised by him, and our analysis complements his work. Our formal model uses a spatial location framework which is related to work by Townsend (1983, 1984). Townsend's models address some of the same concerns as ours. The main differences between our work and his are that our model is analytically more tractable and we use the Nash equilibrium as our main solution concept. We briefly consider the core as an alternative solution concept in the final section of the paper.

In Section I we show that each agent in the economy, faced with uncertain endowments, prefers to trade in a spot market with high rather than low liquidity. Liquidity at a market can only be increased by increasing the number of traders at that market. The traders in our economy are spatially separated so that as more traders go to a particular market, traders are coming from farther away. The farther a trader has to travel to a market, the larger is his transportation cost. Because of this fundamental tradeoff between liquidity and transportation costs, all agents in the economy will not go to the same market.

This paper considers two kinds of competitive market structures. The first assumes that traders may participate in markets without charge (as in standard Walrasian markets). The second structure assumes that it is costly to operate a market, which means that the "auctioneer" must be paid for providing market services. In Section II we establish and characterize the noncooperative symmetric equilibria for the economy with free market services. A basic positive result from this section is that even without fixed cost, liquidity considerations will limit the number of markets in a competitive economy. The welfare implications of the competitive division of markets in this economy are ambigious. Since liquidity is a positive externality, there may be too little liquidity at equilibrium because each agent acts only in his own self-interest. In this case there are too many markets to be efficient. On the other hand, liquidity is self-reinforcing. Given an existing market structure, new markets may find it impossible to open because nobody wants to use a new market with low liquidity. There may be fewer markets than is necessary for efficiency, and yet new markets will not open.

In Section III we make it costly to operate a market. Each market must be operated by a *market maker* whose opportunity cost is the expected utility that he can get as a trader. Arbitrage between being a market maker or a trader reduces the number of equilibria relative to the noncooperative equilibrium with free market services. However, competition between market makers for customers is not sufficient to internalize the externality caused by liquidity. This result runs counter to that of Frank Knight (1924) who suggested that profit-maximizing ownership of a congested facility leads to efficient pricing.[3]

In Section IV we study the market structure when all markets are organized and run by a monopoly exchange. We show that the monopolist will overcrowd the space with small markets.

Liquidity in our model is not tied to the spot market specification in the economy. In Section V we show that the same issues arise when the spot markets are replaced by state-contingent claims between agents at a specific market location. State-contingent claims markets cannot reduce the intrinsic uncertainty of any specific market location. Only the addition of traders at a specific market location may reduce the uncertainty at that location. Since contingent claims markets cannot reduce transportation costs, the same issues remain. In Section V we also briefly consider the concept of the core as an alternative equilibrium concept. Final remarks are also in this section.

[3]Exceptions to Knight's result have been noted elsewhere. For references and a study of duopoly pricing of congested facilities, see Ralph Braid (1986).

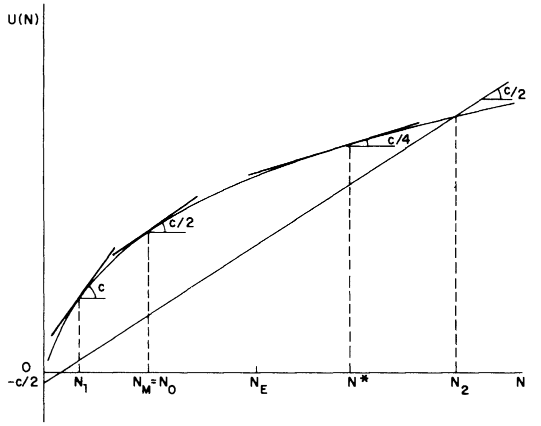

FIGURE 1. LOCATIONS OF AGENTS 1,2,..., N AND MARKETS m_1 AND m_2

I. The Model

Let consumers be located at unit distances apart on the real line (see Figure 1). Each consumer receives at his location a stochastic endowment of two goods x and y. The endowment consists of commodity vector (1,0) with probability $1-\theta$ and of commodity vector (0,1) with probability θ. Consumers have identical preferences over the lotteries of goods x and y received at their location. If a trader goes to another location to exchange goods, the utility of the bundle of goods at the new location is reduced by the disutility from traveling that a trader has to incur to reach the new location.

Consumers, who are expected utility maximizers, can meet each other at any location to exchange goods. The locations at which they meet are called markets.[4] The decision whether to participate in a market is made before the endowment vector is realized. If the consumer does not participate, he consumes his endowment and receives a utility of zero. If he participates in a market, his endowment is realized after he arrives at the market. After the endowments are realized, all consumers in the same market may trade with one another in a competitive spot market for the two goods. Since he has to make his participation decision before he learns of his endowment, a trader evaluates his return from participation by calculating his expected utility from participation. We will show that *ex ante* the expected price at a market is independent of the size of the market, and that the variance of price is lower at a market of high liquidity (with many traders). Thus, a consumer prefers to be in a market of high liquidity.

A consumer incurs disutility from traveling to the market, so that, *ceteris paribus*, he prefers a market closer to his location on the line. Large participation in a market requires that some consumers travel a great distance. Thus, there is a tradeoff between market liquidity and distance between markets. This tradeoff determines the equilibrium distribution of markets in the economy.

When endowments are realized, a consumer may receive one unit of x and zero y, in which case we call him of type 1. Alternatively, a consumer may receive zero units of x and one unit of y, and he is called of type 2. Let a market of N participants consist of X traders of type 1 and Y traders of type 2. Let $k = Y/N$ be the proportion of type 2 traders. A realized market can be described by the pair (k, N). Y is distributed binomially (N, θ). It follows that $E(k|N) = E(Y/N|N) = \theta$.

Assume that exchange in every realized market is Walrasian. Aggregate supply and demand for each commodity are proportional to N, so that the equilibrium price is independent of N.[5] Let $V_1(k)$ (respectively,

[4]The spatial separation of traders on a line has an obvious geographic interpretation. It can also be thought of as measuring time, in the framework of trading in futures' contracts. Then the original location of a trader is interpreted as the date when a trader will receive an uncertain endowment vector. Positions where traders meet (markets) are interpreted as the maturity dates of futures' contracts. Travel costs then represent the utility lost in storage costs covering the time interval between the revelation of endowment and the fulfillment of the futures contract.

[5]Call $x_i(P)$ the demand of a type i trader, where P is the price of y relative to x. Market clearing is defined by $Xx_1(P)+Yx_2(P)=X \Leftrightarrow .\ (1-k)x_1(P)+kx_2(P)=1-k$, which defines a price $P(k)$ independent of N, the size of the market.

$V_2(k)$) denote the indirect utility of a trader of type 1 (respectively of type 2) in a market (k, N). Conditional on a particular value of k, the expected utility of a trader who does not know his type is

$$(1)\quad W(k) \equiv (1-k)V_1(k) + kV_2(k).$$

Since k is a random variable, the unconditional expected utility of a trader in a market of N traders ($N \geq 1$) is

$$(2)\qquad U(N) = E\{W(k)|N\}.$$

For large N, $W(k)$ can be approximated by a Taylor expansion up to the second order around $k = \theta$:

$$(3)\quad U(N) = E\{W(k)|N\}$$
$$\simeq E\{[W(\theta) + (k-\theta)W'(\theta)$$
$$+ (k-\theta)^2 W''(\theta)/2]|N\}$$
$$= W(\theta) + W''(\theta)\theta(1-\theta)/(2N),$$

since $E(k|N) = \theta$, $E\{(k-\theta)^2|N\} = \text{var}(k) = \theta(1-\theta)/N$.[6] For the remainder of this paper, we assume that consumers have either Cobb-Douglas or CES utility functions. As shown in Appendix A, this implies that $W(k)$ is concave and $W''(\theta) < 0$.[7]

$U(N)$, the benefit of a trader from participating in a market of N traders, is an increasing and concave function of N.[8] A trader prefers to be in a larger market, but the marginal advantage decreases as the market becomes larger. Traders prefer larger markets because they provide higher liquidity through lower price variance. However, a trader has to travel from his location to the market to be able to participate in it. The disutility of travel must be subtracted from $U(N)$ to determine the net benefit of participation. We assume that the disutility of travel is linear in the distance traveled at rate c per unit of distance traveled. Thus, a trader participating in a market of N traders at distance α from his original location has net benefit $U(N) - c\alpha$.

II. Noncooperative Equilibria with Free Market Services

We can now look for an equilibrium with free market services. Let agents $j = 1,\ldots$ be located on a real line at consecutive positions one unit distance apart. A strategy of an agent is his choice of final location on the line. He can stay at his initial location and consume his endowment (earning zero utility), or travel to another location on the line in order to trade with other agents. The utility an agent attains by participating in a market depends on how many other agents participate in the same market.

A market structure is a noncooperative equilibrium of the game of market participation. At a noncooperative equilibrium, every agent chooses to travel to a location that maximizes his expected utility under the expectation that all other agents will not change their decision with respect to their market affiliation.

There are many equilibria in this game, including quite unreasonable ones. For example, there is an equilibrium in which everybody stays home because everybody expects all others to stay home. We restrict our attention to symmetric equilibria where every trader participates in a market.

DEFINITION: *At a symmetric equilibrium the distance between neighboring markets is N, and there are N participants per market. Each trader chooses to travel to a location that maximizes his expected utility assuming that all other traders will not change the locations to which they travel.*

[6] Since Y is binomial (N, θ), $\text{var}(k) = \text{var}(Y/N) = (1/N^2)\text{var}(Y) = \theta(1-\theta)/N$. The approximation of equation (2) is good for large N.

[7] The concavity of $W(k)$ can also be derived by allowing agents to participate in location-specific state contingent markets, where we only assume agents have concave utility functions. See Section V.

[8] $dU(N)/dN = -W''(\theta)\theta(1-\theta)/(2N^2) > 0$, $d^2U/dN^2 = W''(\theta)\theta(1-\theta)/N^3 < 0$.

In a symmetric configuration with N traders in each market, we have to make sure that traders prefer to go to the market closest to them. With reference to Figure 1, let market m_1 be located in $1/2$ and market m_2 be located at $N+1/2$. Between these two markets, there is a consumer initially located at every integer from 1 to N. Consider the marginal consumer of market m_1, at distance $(N/2-1/2)$ from m_1 and distance $(N/2+1/2)$ from m_2. He weakly prefers to participate in market m_1 rather than in m_2 if

$$U(N)-c(N-1)/2 \geq U(N+1)-c(N+1)/2,$$

or equivalently

$$U(N+1)-U(N)\leq c.$$

This is approximated by

$$U'(N)\leq c. \tag{4}$$

Let N_1 be the minimal N obeying this inequality,[9] so that inequality (4) is satisfied for all $N\geq N_1$ (see Figure 2). Given that the marginal consumer prefers to go to market m_1 rather than m_2 (under the expectation that all consumers between him and m_1 to go to m_1), any other consumer i closer to m_1 also prefers to go to m_1 rather than m_2 (under the expectation that all other consumers between $1/2$ and $(N/2-1/2)$ go to m_1). This comes directly from the concavity of $U(N)$ in N.

From inequality (4) we see that a necessary condition for the existence of a symmetric equilibrium is that each market has at least N_1 participants. Thin markets located close to one another cannot constitute an equilibrium.

For the equilibrium to exist, we also require that the marginal trader at distance $(N-1)/2$ from m_1, be better off by participating in the market rather than staying home, that is,

$$U(N)-c(N-1)/2\geq 0. \tag{5}$$

Let N_2 be the solution of (5) as equality. All $N\leq N_2$ satisfy inequality (5). Markets cannot be too large because traders from afar do not wish to participate. Inequalities (4) and (5) are necessary and sufficient for the existence of a symmetric equilibrium. Thus, all markets of sizes N in $[N_1, N_2]$ are symmetric noncooperative equilibria. For very large c, $N_2 < N_1$ so that there is no equilibrium.[10]

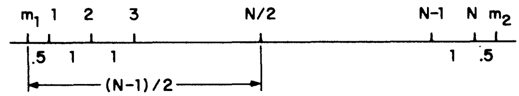

FIGURE 2. MARKETS OF SIZES N IN (N_1, N_2) ARE NONCOOPERATIVE EQUILIBRIA. N^* IS THE OPTIMAL MARKET SIZE. N_M IS THE MONOPOLY MARKET SIZE. N_E IS THE EQUILIBRIUM MARKET SIZE WITH INDEPENDENT MARKET MAKERS

PROPOSITION 1: *Symmetric noncooperative equilibrium market structures exist for disutility cost per unit of distance c that is not prohibitively high. Typically there are many such equilibria characterized by the number of traders per market N, which lies in the interval* $[N_1, N_2]$.

We now consider social welfare in this economy. Let the planner's problem be to set up markets so as to maximize the sum of the expected utility of all traders reduced by the transportation costs they incur. In a market of N participants the sum of the expected utilities of all participants minus transportation costs is

$$S(N)=NU(N) -2c(1/2+3/2+\cdots+(N-1)/2).$$

[9] Using the definition of $U(N)$ in equation (2), N_1 can be calculated as $N_1=[-\theta(1-\theta)W''(\theta)/2c]^{1/2}$.

[10] Let the left-hand side of (5) be defined as $G(N)\equiv U(N)-c(N-1)/2$. $G(N_1)\gtrless 0 \Leftrightarrow N_1 \gtrless N_2$. Using (2), $G(N_1)=W(\theta)+c(1-3N_1)/2$. Since $N_1>1/3$, the term in parentheses is negative. Therefore, $G(N_1)$ will be negative for large c, implying $N_1>N_2$, and thus there will be no N where both (3) and (5) are satisfied simultaneously.

There are L/N markets of N participants in a line of length L. Thus total surplus is

$$s(N) = [NU(N) - 2c(1/2 + 3/2 + \cdots + (N-1)/2)]L/N$$
$$= [U(N) - cN/4]L,$$

proportional to $U(N) - cN/4$, the average utility realized by participants in a market of size N. $s(N)$ is approximately maximized (ignoring integer constraints) at

$$U'(N) = c/4, \qquad (6)$$

the solution of which we call N^*.[11] Comparing (4) with (6) and the concavity of $U(N)$ implies $N^* > N_1$. The result that N^* is larger than N_1 is a consequence of liquidity being a positive externality. When c is low, N^* is lower than N_2 and therefore *the surplus-maximizing solution can be achieved as a noncooperative equilibrium*. This is in contrast with the usual result in price-location models as in Nicholas Economides (forthcoming), and Kelvin Lancaster (1979).

Since N^* belongs in the interval $[N_1, N_2]$, there can also exist equilibria with more or fewer traders per market than is efficient. It is striking that the noncooperative equilibrium in this economy may have more traders per market and fewer markets than the social optimum. This possibility arises because liquidity is self-reinforcing. Given an existing equilibrium, new markets may find it impossible to open because nobody wants to use a new market with low liquidity. There may be too few markets to be efficient, $N_2 > N > N^*$, and yet new markets do not open. So even though liquidity is a positive externality, too much liquidity can result from noncooperative behavior!

When c is large, the social optimum lies beyond N_2 traders per market. To achieve the optimal market structure, traders have to be subsidized to participate in larger and fewer markets. This is the case considered by Diamond (1982).

PROPOSITION 2: *The surplus-maximizing market structure is a noncooperative equilibrium when c is low. Noncooperative equilibrium market size can be larger or smaller than is optimal.*

We now briefly consider the effects of addition of traders to the economy. At first sight it may seem that the addition of traders (say through replication) should decrease the variance of price in every market, increase liquidity, and result in higher expected utility for all traders. In fact, the addition of agents may not reduce the variance of price if the uncertainty is location-specific, so that agents at the same location get identical draws. We show in Appendix B that replicating the number of agents when uncertainty is location-specific leaves liquidity per market constant. Thus, traders' utility is also unaffected. However, replication of the number of agents when uncertainty is not location-specific results in increased liquidity *ceteris paribus*. The resulting equilibrium will have denser markets with increased expected utility for all agents.

III. Competitive Equilibrium with Costly Market Services

In many situations market services are not free. We now allow explicit competition in the provision of market services. Instead of having market services provided free of charge, let each market be operated by a real auctioneer (market maker).[12] Suppose that agents can choose between being market makers or traders. When making this choice they are ignorant of the location they will receive on the line if they decide to be traders. After the choice of occupation, the market

[11] Using the definition of $U(N)$ in equation (2), it is straightforward to show that $N^* = [-2\theta(1-\theta)W''(\theta)/c]^{1/2} = 2N_1$. The fact that we use an infinite L in no way affects the validity of the maximization.

[12] Siow (1982) and Townsend (1983) also studied similar costly financial intermediation.

makers choose their positions on the line to set up their markets. There is no cost to setting up a market except for the opportunity cost of being a trader. The market maker charges each trader a fee F for using his market.[13] After the markets are set up, traders learn their position on the line. Assume that traders are restricted to choose from the set of markets that are offered. Then the traders' decision problem is similar to the one we have discussed before except for the possible difference in fees across markets.

Consider the problem of choice of a fee for the market maker located at m_1. He can lower the fee and hope to attract traders from his competitors, assuming that his competitors will not respond to his price cutting. When he gets additional customers just due to the price cut, his closest competitor to his right, m_2, will lose customers to him. But m_2 will also lose customers to the next market to the right, m_3, because m_2's market is now less liquid. This in turn will drive even more customers away from m_2 toward m_1. Therefore, the number of new customers that m_1 gets from cutting his price depends on the difference in fees and on how liquidity is affected in all other markets. Formally, the new distribution of customers across markets due to m_1's cutting his fee is described by the solution to a second-order difference equation.[14] Market maker m_1 will choose the price that will maximize his profits. But his competitors are doing the same thing, resulting in a Nash equilibrium in fees. Consider the case of symmetrically spaced markets, so that all markets charge the same fee at equilibrium attracting N traders each. Let the equilibrium fee with N traders per market be $F^*(N)$. The wage of a market maker is then $N \cdot F^*(N)$. Since any agent can choose to be a market maker or trader, the wage of the market maker must be equal to the expected utility of a trader from participating in a market of size N. This closes the model and determines the unique market structure in our economy.[15]

We first analyze the game in fees among market makers. After the equilibria of this game are computed, we will return to the (earlier) stage of occupation choice. Formally, let market maker j (operating market m_j) charge a fee F_j for the participation of a trader in his market. For a symmetric positioning of the markets N distance apart, with N traders between every adjacent market, we seek a symmetric noncooperative equilibrium in fees $F^*(N)$.

We now calculate the demand facing market maker $j+1$. Let the marginal trader who is indifferent between going to market m_{j+1} and m_{j+2} be located at $(N_{j+1}-1/2)$ to the right of m_{j+1} (and this implies that he will be $(N-N_{j+1}+1/2)$ away from m_{j+2}). If he goes to market m_{j+1}, there will be $N-N_j+N_{j+1}$ traders in that market and he has to travel $(N_{j+1}-1/2)$ and pay fee F_{j+1}. Therefore, his utility will be $U(N-N_j+N_{j+1})-(N_{j+1}-1/2)c-F_{j+1}$. Similarly, if he goes to market m_{j+2}, there will be $N-N_{j+1}+N_{j+2}+1$ traders at m_{j+2} and his utility will be $U(N-N_{j+1}+N_{j+2}+1)-(N-N_{j+1}+1/2)c-F_{j+2}$. As the trader is on the margin, the following equation has to be satisfied[16]

$$U(N-N_j+N_{j+1})-(N_{j+1}-1/2)c-F_{j+1}$$
$$=U(N-N_{j+1}+N_{j+2}+1)$$
$$-(N-N_{j+1}+1/2)c-F_{j+2}.$$

The system of these equations (j integer) determines the marginal consumers and the demand faced by all market makers as functions of the fees charged.

[13]The fee is in utility units.

[14]We are grateful to Mike Woodford for his help with the difference equations.

[15]Because the number of agents is finite, the demand and profit functions are discontinuous. We thus establish ε equilibria, where market makers optimize up to ε.

[16]This equation approximates the position of the marginal consumer when the N's are treated as integers. Because we are establishing an ε equilibrium, we can treat this relation (and equations (7)–(9) below) as equalities rather than inequalities.

Let all market makers charge the same fee $F_j = F$ except for one market, say F_1, and let $\Delta F = F_1 - F$. Then the positions of the marginal consumers are the solution of the system of equations:

$$(7)\quad U(N - N_j + N_{j+1}) - (N_{j+1} - 1)c$$
$$= U(N - N_{j+1} + N_{j+2} + 1) - (N - N_{j+1})c, \qquad j \neq 0, -1,$$

$$(8)\quad U(N - N_0 + N_1) - (N_1 - 1)c - \Delta F$$
$$= U(N - N_1 + N_2 + 1) - (N - N_1)c, \qquad j = 0,$$

$$(9)\quad U(N - N_{-1} + N_0) - (N_0 - 1)c + \Delta F$$
$$= U(N - N_0 + N_1 + 1) - (N - N_0)c, \qquad j = -1.$$

Defining $\Delta N_j = N_j - N/2$ and linearizing $U(\cdot)$ around $U(N)$ results in

$$(10)\quad \Delta N_{j+2} + \gamma \Delta N_{j+1} + \Delta N_j = \gamma/2, \qquad j \neq 0, -1,$$

$$(11)\quad \Delta N_2 + \gamma \Delta N_1 + \Delta N_0 = \gamma/2 - \Delta F/U', \qquad (j = 0),$$

$$(12)\quad \Delta N_1 + \gamma \Delta N_0 + \Delta N_{-1} = \gamma/2 + \Delta F/U', \qquad (j = -1),$$

where $\gamma = 2(c - U'(N))/U'(N)$.

In Appendix C we solve the system of (10)–(12). Imposing the condition that perturbations at market 1 should have minimal effect at markets far away from m_1, we show that the general solution of (10) with a convergent path is

$$(13a)\quad \Delta N_j = \gamma/2(2+\gamma) + A\rho_1^j, \qquad j = 1, 2, \ldots,$$

$$(13b)\quad \Delta N_j = \gamma/2(2+\gamma) + B\rho_2^j, \qquad j = 0, -1, \ldots,$$

where $\rho_2 = \rho_1^{-1}$ are the roots of the characteristic equation of equation (10) and $\rho_2 < -1 < \rho_1 < 0$ for $U'(N) < c/2$.[17]

Imposing (11) and (12) determines A and B (see Appendix C) as $A = \Delta F/[\rho_1(1 - \rho_1 - \gamma)U']$ and $B = -\Delta F/[(1 - \rho_1 - \gamma)U']$. Thus, demand for firm 1 when it deviates $\Delta F = F_1 - F$ from the fees of all others is

$$N_1 - N_0 + N = \Delta N_1 - \Delta N_0 + N$$
$$= N + 2\Delta F/[(1 - \rho_1 - \gamma)U'].$$

The profit function of firm 1,

$$\Pi_1(F_1) = NF_1 + 2(F_1 - F)F_1/[(1 - \rho_1 - \gamma)U'],$$

is concave in F_1, and is maximized at the solution of

$$4F_1/[(1 - \rho_1 - \gamma)U'] - 2F/[(1 - \rho_1 - \gamma)U'] + N = 0.$$

At the symmetric equilibrium $F_1 = F$, which implies that the equilibrium fee is[18]

$$(14)\quad F^*(N) = N(\rho_1 + \gamma - 1)U'/2.$$

Symmetric equilibrium profits of a market

[17] We could also have considered agent $N_{j+1} + 1$ (at distance $(N_{j+1} + 1/2)$ from m_{j+1}) being indifferent between going to market m_{j+1} or m_{j+2}. Then the resulting system of equations is similar to (10)–(12) except that $\gamma/2$ is replaced by $-\gamma/2$. As seen next in the text, this change only shifts equally the boundaries of the market for any market maker. Therefore, the demand for market maker $j+1$, being

$$N_{j+1} - N_j + N = \Delta N_{j+1} - \Delta N_j + N,$$

remains unchanged and there will be no effect on the equilibrium.

[18] This analysis was done under the assumption that all consumers participate in the market, that is, that $N(\gamma + \rho_1 - 1)U'/2 = F^*(N) \leq U(N) - c(N-1)/2$. $F^*(N)$ is positive since it is proportional and of the same sign as $\rho_1 + \gamma - 1 = [c - 2U' + c^{1/2}(c - 2U')^{1/2}]/U' > 0$ because $U' < c/2$.

maker when markets are N apart are

$$(15)\quad \Pi_E(N) = N^2(\rho_1 + \gamma - 1)U'/2 = N^2\left(c - 2U' + [c^2 - 2cU']^{1/2}\right)/2.$$

This is an increasing function of N.[19]

PROPOSITION 3: *The symmetric fee structure $F_i = F^*(N)$ given by* (14) *is a noncooperative equilibrium of the game among market makers when markets are set N apart. Equilibrium profits are given by* (15).

Now we return to the choice of occupations. The overall equilibrium market structure is determined by the condition that equalizes the expected profits of a market maker with the expected utility of a trader. The expected utility of a trader when he does not know his position on the line (following the analysis of Section III) is $S(N) = U(N) - cN/4 - F^*(N)$, a concave function of N passing through the origin. The equilibrium market structure is determined by the intersection of $\Pi_E(N)$ and $S(N)$. Since $\Pi_E(N)$ is defined for $N > N_0$, where $U'(N_0) = c/2$, the equilibrium N_E exists if $S(N_0) - \Pi_E(N_0) \geq 0$. Equilibrium fees can be calculated by substitution of N_E in (14).

In general, the equilibrium N_E is not the same as the optimal market structure N^*, which is analogous to the one of Section II adjusted for the the market makers' wages. In fact, results for the Cobb-Douglas utility function show that N_E can be smaller or larger than N^*. Thus, even when market makers are aware of the gains from liquidity, and there is competition in fees between market makers and free entry of market makers, the externality caused by liquidity is still not completely internalized. As noted in our introductory discussion, this result is counter to Knight's (1924), where he argued that profit-maximizing ownership of a congested facility leads to efficient pricing.

PROPOSITION 4: *There exists a unique symmetric equilibrium market structure with N_E traders per market, where the ex ante expected utility of an agent is the same in either occupation. Liquidity in this equilibrium may exceed or fall short of the level needed for surplus maximization.*

Note that function $S(N)$ is decreasing in c while $\Pi_E(N)$ is increasing in c.[20] Therefore, the intersection N_E of $S(N)$ and $\Pi_E(N)$ decreases in c. A decrease in the disutility of travel results in fewer and larger markets.

IV. The Monopolist's Solution

So far this paper has only considered competitive market structures. However, many financial markets in the United States, where liquidity considerations are important, are organized by a few financial exchanges. A study of competition between financial exchanges using oligopolistic models of product differentiation is beyond the scope of this paper. As a benchmark, we discuss the market configuration chosen by a monopolist exchange that acts as a market maker in all markets.

The monopolist's problem is particularly relevant for a futures exchange that has to determine the maturity dates of futures contracts in a commodity. Contracts of different maturity dates compete among themselves for liquidity. The extent that a nondiscriminating monopolist can appropriate the market-generated surplus depends on the distance between markets and the level of liquidity in each market. Thus it is a nontrivial choice problem for the monopolist to determine the optimal number of maturity dates and the corresponding fees.

Within the context of our model, the monopolist's objective is to maximize total revenue collected from all markets. Once the monopolist announces the locations of markets and the fee structure, the traders decide whether to participate in a market and in

[19] For convexity it is sufficient that $U''' < 0$.

[20] $d\Pi_E(N)/c = 1 - 2U' + (c - U')/[c^2 - 2cU']^{1/2} > 0$ for $0 \leq c \leq 1$ because the second term is always positive since $U' < c/2 < c$, and the first term is positive if $U' < c/2 \leq 1/2 \leq c \leq 1$.

which market to do so. The monopolist will serve all agents on the line. This is because, for any fee structure that leaves some agents at home, the monopolist can bring the markets closer together, close the gap, and increase revenue by establishing a market in the freed space. Thus, the monopolist wants to make the marginal agent, at distance $c(N-1)/2$, indifferent between participating and staying home. From each market the monopolist collects $F(N)\cdot N$. Since the frequency of the markets is $1/N$, total revenues are proportional to $F(N)$. Thus, the monopolist's problem is to

$$\underset{N}{\text{Maximize}}\ F(N)$$

subject to

$$U(N)-c(N-1)/2-F(N)\geq 0,$$

that is, that traders come to the market rather than stay home, and also subject to

$$U'(N)\leq c, \tag{4}$$

that is, that the symmetric equilibrium fee structure of the monopolist is a noncooperative equilibrium for traders. It is equivalent to

$$\underset{N}{\text{Maximize}}\ U(N)-c(N-1)/2 \tag{16}$$

$$\text{subject to } U'(N)\leq c.$$

Its solution is at N_M defined by

$$U'(N)=c/2. \tag{17}$$

Clearly N_M is in (N_1, N_2). In comparison with the surplus-maximizing outcome, the monopolist will operate a larger number of smaller markets.[21] This is because the surplus-maximizing outcome is defined by $U'(N^*)=c/4$ (equation (6)) and $U(\cdot)$ is concave.

Comparing the monopolist's market structure with the equilibrium of independent market makers of Section IV, we see that $U'(N_M)=c/2>U'(N_E)$. By the concavity of $U(\cdot)$, $N_M<N_E$. The monopolist will open smaller and more numerous markets than independent market makers.

PROPOSITION 5: *A monopolist will operate smaller and more numerous markets than independent competing market makers. Further, his markets are always smaller and more numerous than is optimal.*

Lacking the ability to price discriminate and appropriate the whole surplus, the monopolist avoids creating large markets with high surplus. Instead he institutes a large number of smaller markets where he can appropriate a larger percentage of the surplus. We note that *the overcrowding of the space with markets happens despite the fact that there is no threat of entry*. Overcrowding of the product space to deter potential entrants has been noted by Richard Schmalensee (1978), among others.

V. Discussion

Consider replacing each spot market in this economy with state-contingent claims markets (for example, Debreu, 1959, ch. 7) for agents that go to the same market location. Agents will still go to a specific market location. But each agent now trades in state-contingent claims with other agents at the same location before he knows the realization of endowments. Every agent at the same market location is *ex ante* identical. So every agent will have the same excess demand functions for these contingent claims commodities. The equilibrium prices must be such that all agents will have the same *ex post* consumption bundle. Therefore, when k is realized, each agent at the same market will consume $(1-k, k)$. Assuming that the representative agent has a utility function which is concave in the two goods, his indirect utility function will also be concave in

[21] Using the definition of $U(N)$ in equation (2), it is easy to show that $N^*/N_M=\sqrt{2}$ so that the monopolist operates approximately 40 percent more markets than is optimal.

k.[22] Substituting this indirect utility function in equation (2) in the place of $W(k)$, one can derive an expected benefit function with the same properties as $U(N)$. The results of the rest of the paper follow. Therefore, the qualitative features of the symmetric noncooperative equilibria are the same whether agents are faced with spot markets or state-contingent claims markets.

The difference between the state-contingent claims markets in this economy and in the standard general equilibrium model is that agents in this economy must go to a specific market location before they can participate in the state-contingent claims markets. There is risk sharing within a market location but not across market locations. In the standard model, agents can participate in a complete market structure without first having to go to any specific location. The standard model allows risk sharing among all individuals in the economy, whereas our model only allows risk sharing among endogenous subsets of individuals.

The alternative market setup considered above shows that it is not the spot market setup of our problem that is important. The normative results on liquidity is due to the Nash equilibrium concept that we employ. Some readers have questioned whether alternative equilibrium concepts, such as the core (as used in studies on financial intermediation by John Boyd and Edward Prescott, 1986, and Townsend, 1983), may give different normative results. In our problem, the core is efficient because it allows a redistribution of income from agents close to a market to those who are far away. The redistribution means that all agents who go to the same market will have the same *ex ante* utility, making them indifferent to their distance from the market. The efficient market structure allows for the largest surplus to be redistributed, which means no other coalition can be formed that will satisfy all agents in this other coalition. Therefore, the core will provide the efficient level of liquidity. The main difficulty with using the concept of the core in our problem is that the redistribution that is necessary seems difficult to enact for many relevant problems. In particular, the identity of every agent (that is, his location) has to be common knowledge to all agents in order to implement the core allocation. The identity of agents is not necessary for constructing the Nash equilibria. If the identities of agents were somehow known, it is possible that price discrimination in fees by market makers within the Nash equilibrium construct might mimic the core solution.

An area in which the model with free market services might apply is in the choice of standards when there is a variety of products. For example, the personal computer industry has a large variety of potential and actual products. In this industry a few standards have already arisen. Many consumers buy an IBM or IBM-compatible machine, even though it is not the "best" for what they currently want to do. Other products may be able to do what they want better and at a lower price. However, most consumers know that they may use the computer to solve other problems in the future. By buying "the standard," they are buying insurance that accessories (software and hardware) will be available for solving those problems. On the other side of the market, firms may not produce products which accomplish a task most efficiently, but will rather produce IBM-compatible products. Since firms do not have to pay a fee in choosing the "standard," there will be a range of indeterminancy for the equilibrium standards, as predicted by our model. The standards that obtain in an actual industry can often be predicted by participants in that industry from their knowledge of initial conditions. For example, most informed observers expected IBM to become a standard in the personal computer industry. This observation is not inconsistent with the fact that at some point "the standard" may look

[22] In this setup $W(k) = w(1-k, k)$ where $w(\;,\;)$ is concave. Then under regularity $W''(k) = w_{11} + w_{22} - 2w_{12}$. For concavity of $W(\cdot)$ we need to show $|w_{11}| + |w_{22}| + 2w_{12} > 0$. This is obviously true for $w_{12} > 0$. For $w_{12} < 0$, it is sufficient to show that $|w_{11}| + |w_{22}| - 2|w_{12}| > 0$. By concavity of w, we know that $(|w_{11}w_{22}|)^{1/2} > |w_{12}|$. Thus $|w_{11}| + |w_{22}| - w|w_{12}| > |w_{11}| + |w_{22}| - 2(|w_{11}w_{22}|)^{1/2} = (|w_{11}|^{1/2} - |w_{22}|^{1/2})^2 \geq 0$.

arbitrary, given the available knowledge and technology in that society. Research on network externalities considers closely related issues (for example, Dennis Carlton and Mark Klamer, 1983; Joseph Farrell and Garth Saloner, 1985; Michael Katz and Carl Shapiro, 1985). Recent work (Economides, 1987; Carmen Matutes and Pierre Regibeau, 1986) has also demonstrated that firms may have incentives to produce compatible products, even in the absence of network externalities.

Another example where free standards matter is the agents' hours of work in an economy. Workers have to interact with other workers and clients both within and outside the firm. Often these interactions are not mutually anticipated or coordinated. Our model suggests that workers with heterogeneous preferences will tend to work the same schedule. Moreover, the comparative statics results in Appendix B suggest that there will be more numerous and diverse types of work schedules in a large city, where there is a larger number of types of workers and more workers of each type. Empirical evidence in Siow (1987) shows that a worker is penalized for not coordinating his hours of work with his co-workers.

APPENDIX A

A solution of the problem when agents have Cobb-Douglas utility functions follows. Let P be the relative price of good y with respect to x. A consumer with utility function $U(x, y) = x^{\alpha}y^{\beta}$, when endowed with A units of x, has budget constraint $x + Px = A$ and (gross) demands $x_1 = \alpha A/(\alpha+\beta)$, $y_1 = \beta A/(\alpha+\beta)P$. When endowed with A units of y he has budget constraint $x + Py = AP$ and demands $x_2 = \alpha AP/(\alpha+\beta)$, $y_2 = \beta A/(\alpha+\beta)$. Market clearing implies $x_1 X + x_2(N - X) = XA \Leftrightarrow P = \beta X/\alpha(N - X) = \beta(1-k)/\alpha k$. The equilibrium indirect utility function of a consumer endowed with A units of x participating in a market (N, k) is $V_1(k) = (\alpha A/(\alpha+\beta))^{\alpha+\beta}(k/(1-k))^{\beta}$. For a consumer endowed with A units of y, the corresponding indirect utility is $V_2(k) = (\beta A/(\alpha+\beta))^{\alpha+\beta}((1-k)/k)^{\alpha}$. Let $W(k) = (1-k)V_1(k) + kV_2(k)$ as in equation (1) in the text. A straightforward calculation will show that $W(k)$ is concave as long as α, β are in the open interval (0,1). When the utility functions are CES, $U(x, y) = (x^{\rho} + y^{\rho})^{1/\rho}$, $\rho < 1$, a type 1 consumer, facing constraint, $x + Py = A$, will demand $x_1 = A/(1 + P^r)$, $y_1 = AP^{r-1}/(1+P^r)$, where $r = \rho/(\rho - 1)$. Similarly a type 2 consumer, facing constraint $x + Py = AP$, will demand $x_2 = AP/(1+P^r)$, $y_2 = AP^r/(1+P^r)$. Market clearing implies $x_1 X + x_2(N - X) = XA \Leftrightarrow$

$$\text{(A1)} \qquad 1 - k + kP = (1-k)(1+P^r) \Leftrightarrow P = (k/(1-k))^{1/(r-1)}.$$

The indirect utility functions are $V_1 = A(1+P^r)^{-1/r}$, $V_2 = AP(1+P^r)^{-1/r}$. Thus, $W(k) \equiv (1-k)V_1 + kV_2 = (1-k+kP)V_1 = (1-k)A(1+P^r)^{1-1/r}$, using (A1). Substituting the clearing price we have

$$W(k) = A(1-k)\left[1 + (k/(1-k))^{r/(r-1)}\right]^{(r-1)/r}$$

Direct computation reveals

$$W'(k) = A\left[1 + (k/(1-k))^{r/(r-1)}\right]^{-1/r} \times \left[-1 + (k/(1-k))^{1/(r-1)}\right],$$

and

$$W''(k) = A(k/(1-k))^{1/(r-1)} \times \left[1 + (k/(1-k))^{r/(r-1)}\right]^{-1-1/r} / \left[(r-1)k(1-k)^2\right] < 0,$$

where all terms except the denominator are positive. $r - 1$ is negative for all $\rho < 1$, that is, for the whole range of definition of the CES.

APPENDIX B

Suppose we double all agents at the old positions. First, let us consider the case when uncertainty is not location-specific. Let any two agents at the same location get independent draws from the distribution of endowments. At every market, the distribution of types is preserved. Given k, the equilibrium price is unaffected and so are the indirect utility functions $V_1(k), V_2(k)$ of each trader and their weighted sum $W(k)$. The *function* $U(N)$ is the same as before, but now it has to be evaluated at $2N$. Let markets be $\hat{N}_1 d$ apart, at the lower bound of the equilibrium existence region (where the corresponding number before doubling was $N_1 d$). There are now $2\hat{N}_1$ traders per market. Equation (4) for this equilibrium is

$$\text{(A2)} \qquad U'(2\hat{N}_1) = cd,$$

which implies $2\hat{N}_1 = N_1$, or $\hat{N}_1 = N_1/2$, since N_1 solves $U'(N_1) = cd$. Thus, when agents are doubled at the old locations and receive independent draws, the number of agents per market remains unaffected ($2\hat{N}_1 = N_1$), while the markets are twice as dense at distances $\hat{N}_1 = N_1 d/2$ (compared with $N_1 d$ originally). The expected utility of agents is now higher because they have to travel half the distance to find a market of the same liquidity as before doubling. Alternatively, suppose that all uncertainty is location-specific, such as uncertainty associated with the local weather. After replication, let agents receive identical endowments. Again $V_1(k)$, $V_2(k)$, and $W(k)$ are unaffected. Further, the variance of k is the same as before replication, although there are now twice as many agents per market. The original expected utility function

$$\text{(A3)}\quad U(N)=W(\theta)+W''(\theta)\theta(1-\theta)/(2N)$$

has been replaced by a new utility function for the replicated model

$$U_1(2N)=W(\theta)+W''(\theta)\theta(1-\theta)/(2N),$$

and therefore

$$U_1(N)\equiv U(N/2).$$

Let markets be $\hat{N}_1'd$ apart at the lower bound of the equilibrium existence region. There are now $2\hat{N}_1'$ traders per market. Equation (4) is now

$$U_1'(2\hat{N}_1')=cd\Leftrightarrow U'(\hat{N}_1')=cd\Leftrightarrow \hat{N}_1'=N_1.$$

The resulting markets are at the same distance as before replication, with twice as many traders per market, but all traders receive the same utility as before, $Z_1(2N,\alpha)=U_1(2N)-c\alpha=U(N)-c\alpha=Z(N,\alpha)$.

We have shown that replicating the number of agents when uncertainty is location-specific leaves liquidity per market constant. Thus, traders' utility is also unaffected. However, replication of the number of agents when uncertainty is not location-specific results in increased liquidity *ceteris paribus*. The resulting equilibrium will have denser markets with increased expected utility for all agents.

APPENDIX C

The homogeneous version of equation (10), $\Delta N_{j+2}+\gamma\Delta N_{j+1}+\Delta N_j=0$, has the characteristic equation $\rho^2+\gamma\rho+1=0$ with solutions $\rho_1=(-\gamma+(\gamma^2-4)^{1/2})/2$, $\rho_2=1/\rho_1$, or equivalently, since $\gamma=2(c-U')/U'$, $\rho_1=(U'-c+[c^2-2cU']^{1/2})/U'$, $\rho_2=(U'-c-[c^2-2cU']^{1/2})/U'$. The roots are real distinct for $U'<c/2$. For $U'=c/2$, they are real and coinciding $\rho_1=\rho_2=-1$. Then the solution of the homogeneous equation $\Delta N_j=A(-1)^j+Bj(-1)^j$ diverges as j goes to infinity. For $U'>c/2$, the roots are complex and the solution is $\Delta N_j=A\cos(\theta j)+B\sin(\theta j)$, where $\cos\theta=-\gamma/2=(U'-c)/U'$, an exact oscillation. In the last two cases a disturbance at $j=1$ has large effects at markets far away, an event we rule out. In the case of distinct roots arising from $U'<c/2$, it is easy to see that $\rho_2<-1<\rho_1<0$. The general solution of the homogeneous equation for markets to the right of m_1, $\Delta N_j=A_1\rho_1^j+P_2\rho_2^j$, converges as $j\to\infty$, if and only if A_2 is zero. Similarly, the general solution for markets to the left of m_1, $\Delta N_j=B_1\rho_1^j+B_2\rho_2^j$, converges as $j\to-\infty$, if and only if B_1 is zero. Hence the convergent solution is

$$\Delta N_j=A\rho_1^j,\qquad j=1,2,\ldots,\text{ and}$$

$$\Delta N_j=B\rho_2^j,\qquad j=0,-1,\ldots.$$

The inhomogeneous equation (10) has a particular solution $\Delta N_j=\gamma/2(2+\gamma)$.

Thus the solution of (10) is

$$\Delta N_j=\gamma/2(2+\gamma)+A\rho_1^j,\qquad j=1,2,\ldots,$$

$$\Delta N_j=\gamma/2(2+\gamma)+B\rho_2^j,\qquad j=0,-1,\ldots.$$

Conditions (11) and (12) can thus be written as

$$A\rho_1^2+A\gamma\rho_1+B=-\Delta F/U'$$

$$A\rho_1+B\gamma+B\rho_1=\Delta F/U',$$

which are solved by $A=\Delta F/[(1-\rho_1-\gamma)U'$, $B=-\Delta F/[(1-\rho_1-\gamma)U']$, so that the solution of the system is

$$\Delta N_j=\gamma/2(2+\gamma)+\Delta F\rho_1^{j-1}/[(1-\rho_1-\gamma)U'],\qquad j=1,2,\ldots$$

and

$$\Delta N_j=\gamma/2(2+\gamma)-\Delta F\rho_2^j/[(1-\rho_1-\gamma)U'],\qquad j=0,-1,\ldots.$$

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