NOTES FOR MICROECONOMICS

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MICROECONOMICS is about

1. Buying decisions of the individual

2. Buying and selling decisions of the firm

3. The determination of prices and in markets

4. The quantity, quality and variety of products

5. Profits

6. Consumers' satisfaction

There are two sides in a market for a good

DEMAND	**SUPPLY**
Created by Consumers	Created by firms
Each consumer maximizes	Each firm maximizes its
satisfaction ("utility")	profits
------------------------------	------------------------------
↑ CONSUMPTION THEORY	↑ PRODUCTION THEORY

We will first study consumption and later production. In the third part of the course we will take the "demand" schedule from the consumption analysis and the "supply" schedule from the production analysis and put them together in a **market**. The price, and the quantity exchanged will be determined in the market. We will also discuss the performance and efficiency of markets.

A.   CONSUMPTION ANALYSIS UNDER CERTAINTY

1. **Goods** are products or services that consumers or businesses desire. Examples: a book, a telephone call, insurance coverage. Goods may be directly desired by consumers or may contribute to the production of other goods that are desired by consumers. For example a machine used in the production of cars is desirable because it is useful in the production of cars, although it has no direct value to a consumer. **Bads** are products or services that consumers desire *less of*. Examples: garbage, pollution, some telephone calls. Clearly, a good for one consumer could be a bad for another.

2. If possible, each consumer would consume a very large (infinite) amount of each good. But, each individual is constrained by his/her ability *to pay for these goods*. The limitation of total funds available to an individual defines the

budget constraint. Therefore a consumer has to maximize his/her satisfaction while not spending more than he/she has, *i.e.*, without violating the budget constraint.

3. We are interested to find the best choice for a consumer that has a limited amount of funds. We accomplish this in three steps. At the first step, we define the available choices taking into account the limitation of funds. At the second step, we discuss the desires/wants of the consumer. At the third step, we find the optimal choice for the consumer by putting together the information we gathered in the previous two steps.

STEP 1: We first analyze the available choices to a consumer that possesses limited funds. Suppose there are only two goods, X and Y, and they are sold at prices p_x and p_y per unit respectively. If a consumer buys x units of good X and y units of good Y, she spends xp_x on good X, and yp_y on good Y. Total **expenditure** is

$$E = xp_x + yp_y.$$

The pair (x, y) is called a (consumption) **basket** or (consumption) **bundle**.

If the consumer has a total amount of money I (income) her total expenditure cannot exceed I, *i.e.*,

$$xp_x + yp_y \leq I.$$

This is called the **budget constraint**. The set of available (x, y) combinations is
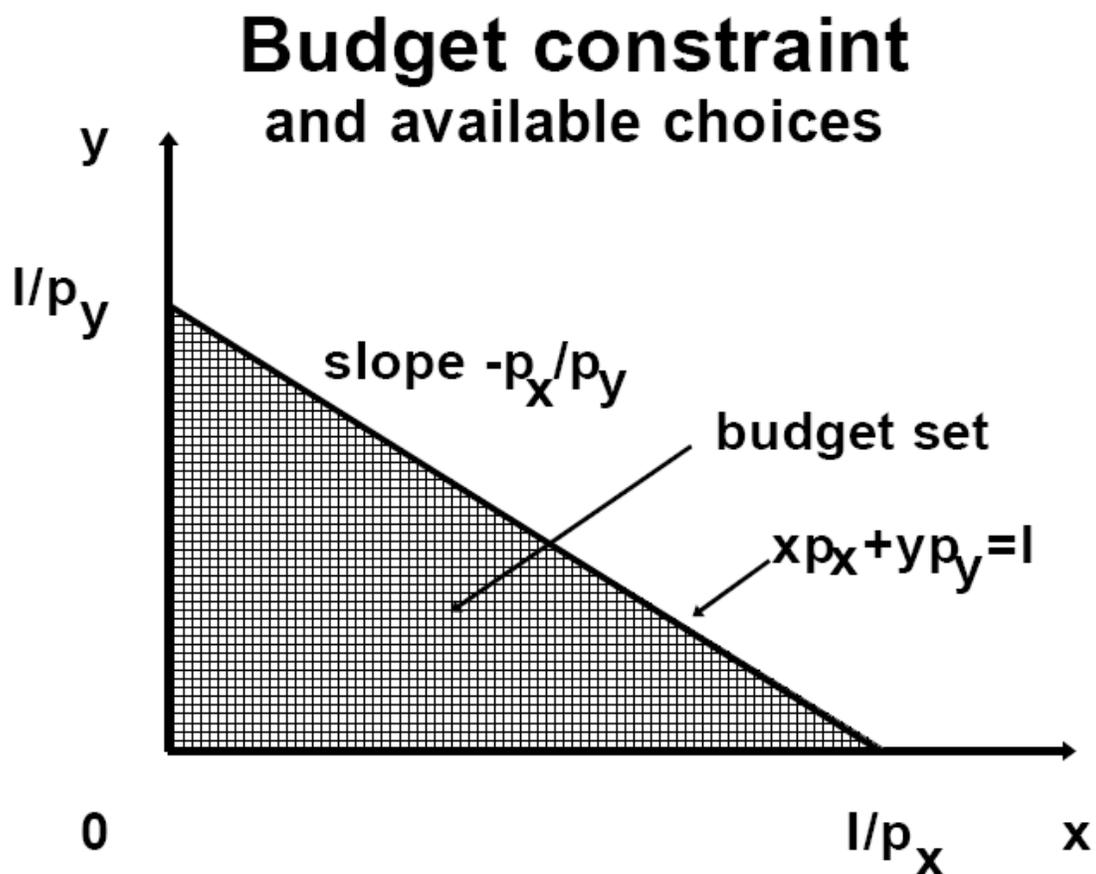
called the **budget set**. See Figure 1.

Budget constraint and available choices

y

I/p_y

slope $-p_x/p_y$

budget set

$xp_x + yp_y = I$

0

I/p_x

x

Figure 1

For example, X is apples sold at $1 per pound, Y is oranges sold at $0.5 each,

and the consumer has I = $3 to spend. Then the basket (1, 4) (*i.e.*, 1 pound of

apples and 4 oranges) costs 1 + 4(.5) = $3, and therefore is in the budget set.

Note that since basket (1, 4) available to the consumer, so are baskets where she buys less of each or less of both of the goods, such as (0, 4), (1, 3), (1, 2), etc., since they cost less.

4. The budget constraint can be represented in the X-Y space. There are two cases, either the consumer spends all her income and $xp_x + yp_y = I$, or the consumer has some left-over income and $xp_x + yp_y < I$. We are primarily interested in the case where all income is spent. Note that and $xp_x + yp_y = I$ is a straight line in the X-Y space. It is called the **budget line**. Its slope is $-p_x/p_y$. If the consumer spends all the money in good X then she buys I/p_x units of this good. This is the maximum amount of X that she can buy. It defines the most extreme point of the triangle on the X-axis. If all money is spend on good Y, it will buy I/p_y units of good Y. This is the most extreme point of the triangle on the y-axis.

All bundles (x, y) where the consumer does not exhaust all income are below the budget line. The budget set contains all the points in the shaded triangle, including its boundary lines.

STEP 2: We now describe the preferences of the consumers.

5. In a comparison of *any* two **bundles**, $A = (x_A, y_A)$ and $B = (x_B, y_B)$, an

individual should be able to say either

(i) "I **prefer** A to B"; or

(ii) "I **prefer** B to A"; or

(iii) "I am **indifferent** between A and B", *i.e.*, "I like equally A and B".

This property of preferences is called **completeness**. Essentially the consumer is not allowed to say "I don't know" or "I am not sure."

The second property of preferences is **transitivity**. If a person states, "I prefer A to B," as well as "I prefer B to C," then he/she also *has to* prefer A to C. This assumption says that preferences are consistent, so that comparisons between bundles A and C are consistent with comparisons between bundles A and B and between B and C.

Transitivity in indifference means that a person who says, "I am indifferent between A and B," as well as "I am indifferent between B and C," also *has to* be indifferent between A and C.

The third assumption on preferences is that "more is better".

6. Since "more is better", if bundle $A = (x_A, y_A)$ has more of both goods than bundle $B = (x_B, y_B)$, *i.e.*, if $x_A > x_B$ and $y_A > y_B$, then clearly a consumer will prefer A to B. Similarly if bundle A has less of both goods than bundle B, then

a consumer will prefer B to A. However, if bundle A has more of X but less of Y than B, the comparison is not obvious. The consumer may prefer A to B, or prefer B to A, or be indifferent between A and B.

One can create a collection of all the bundles, A, B, C, D, ..., such that a *particular* consumer is indifferent between any two of them. The line in X-Y space that connects the points in this collection {A, B, C, D, ...} is called an **indifference curve**, I_1 (Figure 2). Of course, the same consumer typically has many indifference curves. For example, he has both A-B-C-D and E-F-G-H as indifference curves, and he prefers any bundle on E-F-G-H to any on A-B-C-D. In general, there is an indifference curve through any point in X-Y space. Since "more is better," an indifference curve cannot have a positive slope. Indifference curves have a negative slope, and in special cases zero slope. An indifference curve defines the substitution between goods X and Y that is acceptable in the mind of the consumer. As we move towards the Southeast along a typical indifference curve the consumer receives more X and less Y, while she declares that she is equally well off.
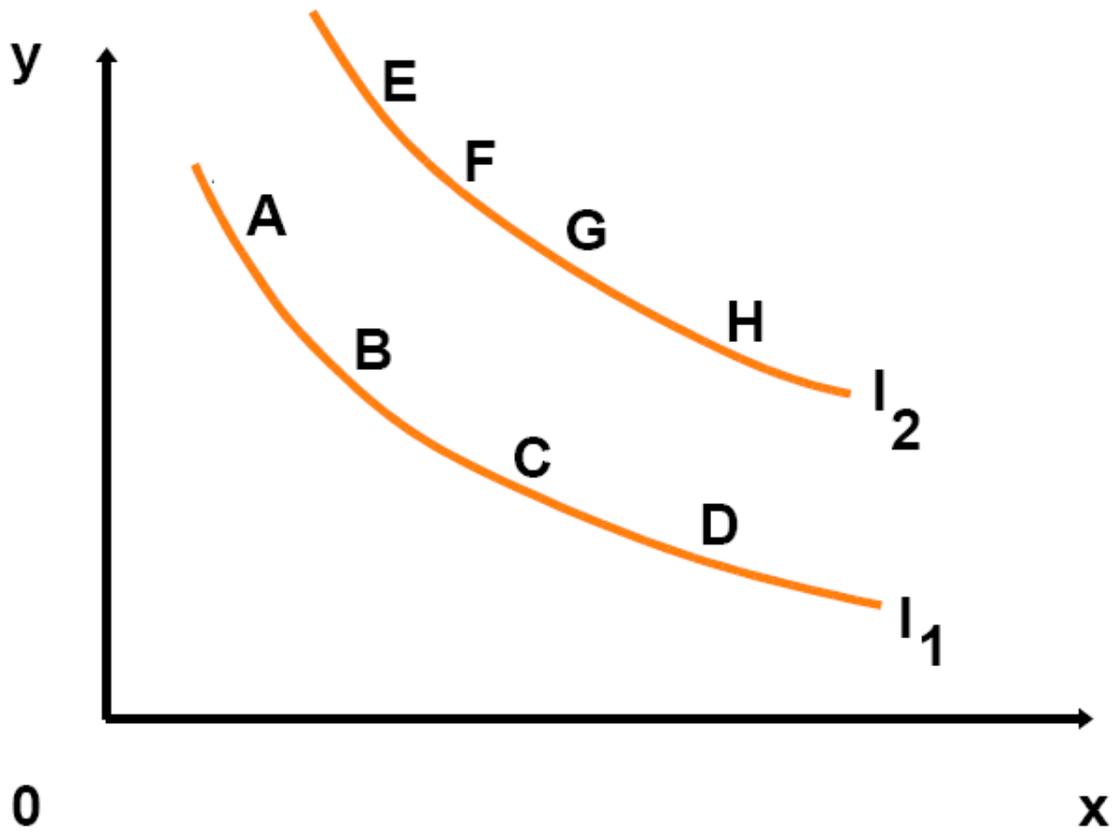
Indifference curves

Figure 2

7. Typically indifference curves are **convex**. This means that starting with two bundles, A, B, which the consumer the consumer likes equally and are therefore on the same indifference curve, she prefers C, the average of two extreme bundles, rather than either of them. Bundle C has the average quantities of bundles A and B in X and Y (Figure 3).
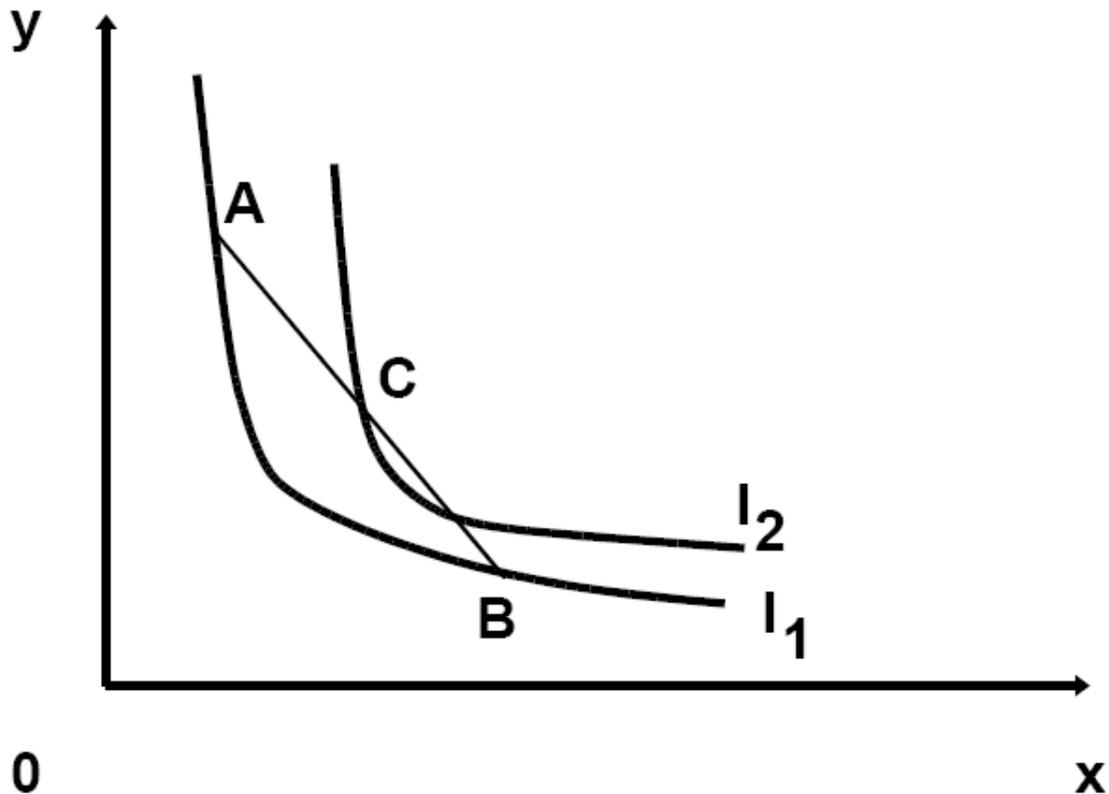
Convex indifference curves

Figure 3

8. **Special cases** of indifference curves. If the goods are **perfect substitutes**, the indifference curves have a constant slope, *i.e.*, are straight lines. This means that the substitution between good X and good Y is constant, irrespective of the point on the indifference curve. Example, X is nickels, Y is dimes (Figure 4). If the goods are **perfect complements**, the consumer combines the goods in a fixed proportion. Then, indifference curves are L-shaped. <u>Examples:</u> X is left

shoes, Y is right shoes; X is personal computer CPUs, Y is video monitors
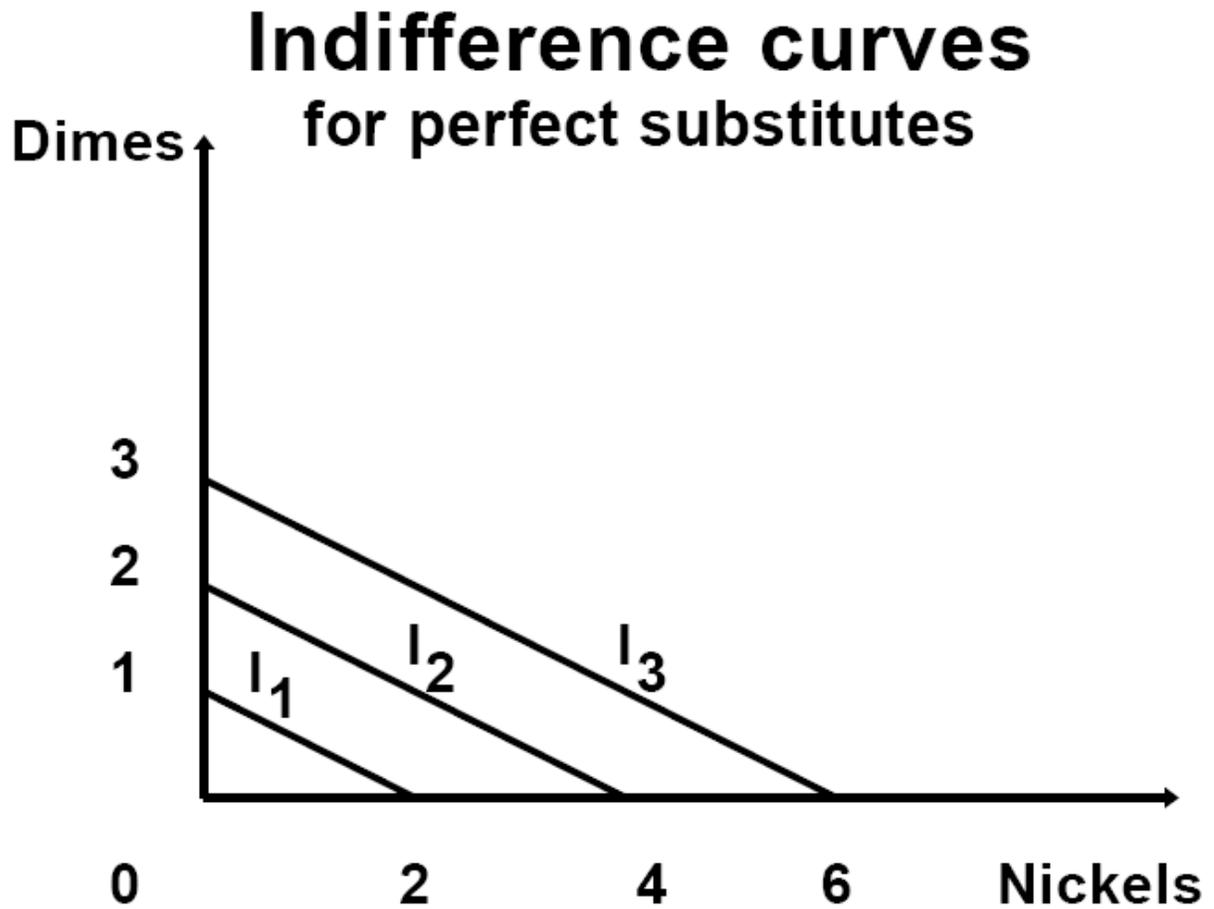
(Figure 5).

Indifference curves for perfect substitutes

Dimes

3

2

1

I_1 I_2 I_3

0   2   4   6   Nickels

Figure 4
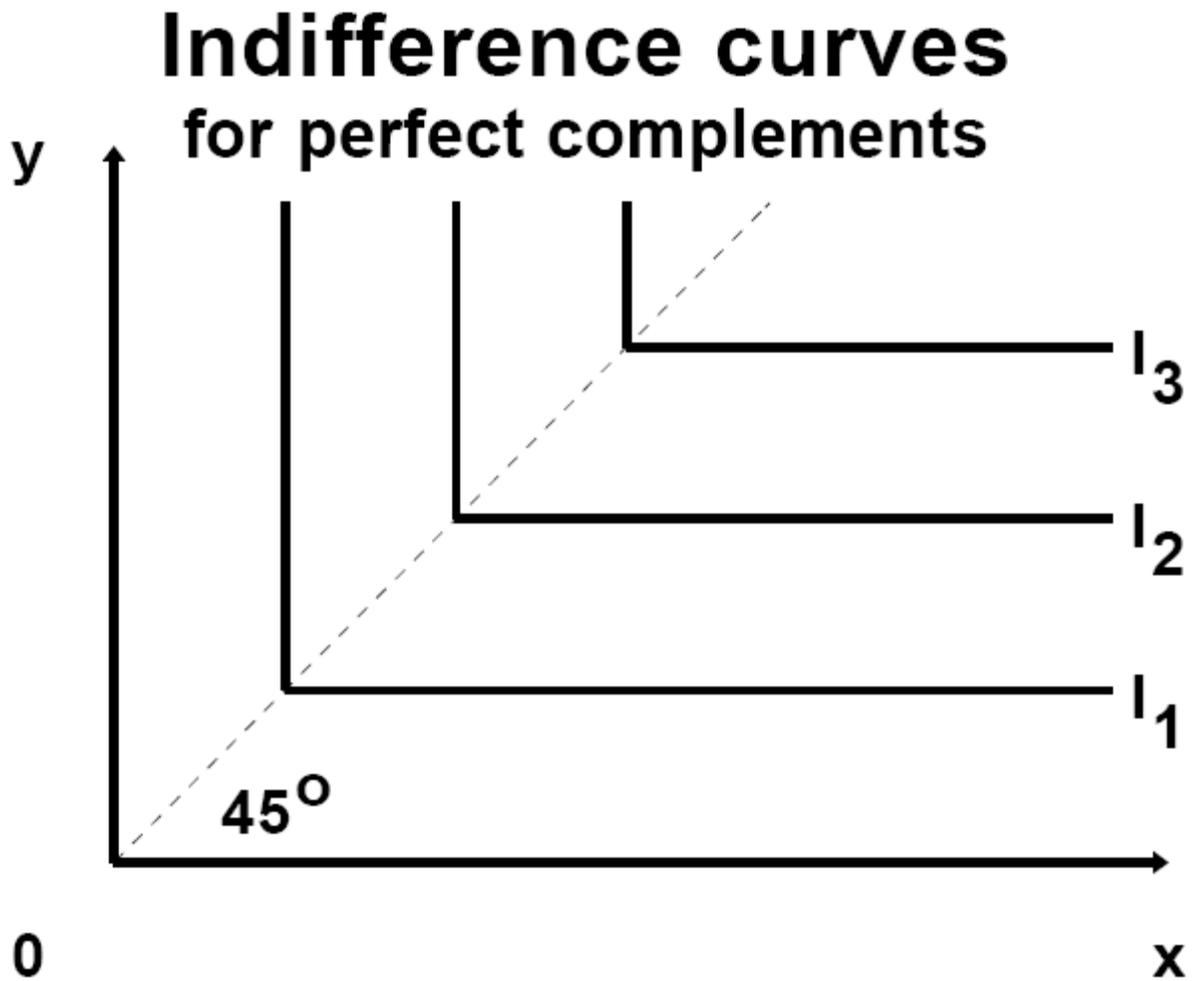
Indifference curves
for perfect complements

Figure 5

9. A consumer can be thought of as assigning a level of **satisfaction** (or **utility**) to each bundle, utility of bundle (x, y) is U(x, y). Then, all bundles on the same indifference curve give the same level of satisfaction (utility). For example, in our earlier figure, the level of satisfaction of a consumer may be 2 at any point on indifference curve EFGH, and the level of satisfaction may be 1 at any point

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on indifference curve ABCD.

10. *The **marginal rate of substitution**, is the rate at which a consumer is willing to trade x for y. It is the slope of an indifference curve, MRS = Δy/Δx.* In general, the MRS varies along an indifference curve, that is, the MRS is in general different when the starting bundle of a potential trade changes. For *perfect substitutes*, the MRS is constant. Note that the marginal rate of substitution (MRS) of consumer Z depends on individual preferences as expressed by the indifference curves. It does *not* depend on the market or the prices that may prevail in the market.

11. An additional unit of good X increases the level of satisfaction of a consumer by the **marginal utility of X**, MU_x. Similarly, an additional unit of Y increases the level of satisfaction of a consumer by the **marginal utility of Y**, MU_y. *The marginal rate of substitution is equal to the ratio of the marginal utilities,*

$$\mathbf{MRS = \Delta y/\Delta x = - MU_x/MU_y}$$

Why? In Figure 6, consider a move from bundle A to bundle B on the same indifference curve. It can be broken into a vertical piece (change in Y) and a horizontal piece (change in X) by defining bundle C.
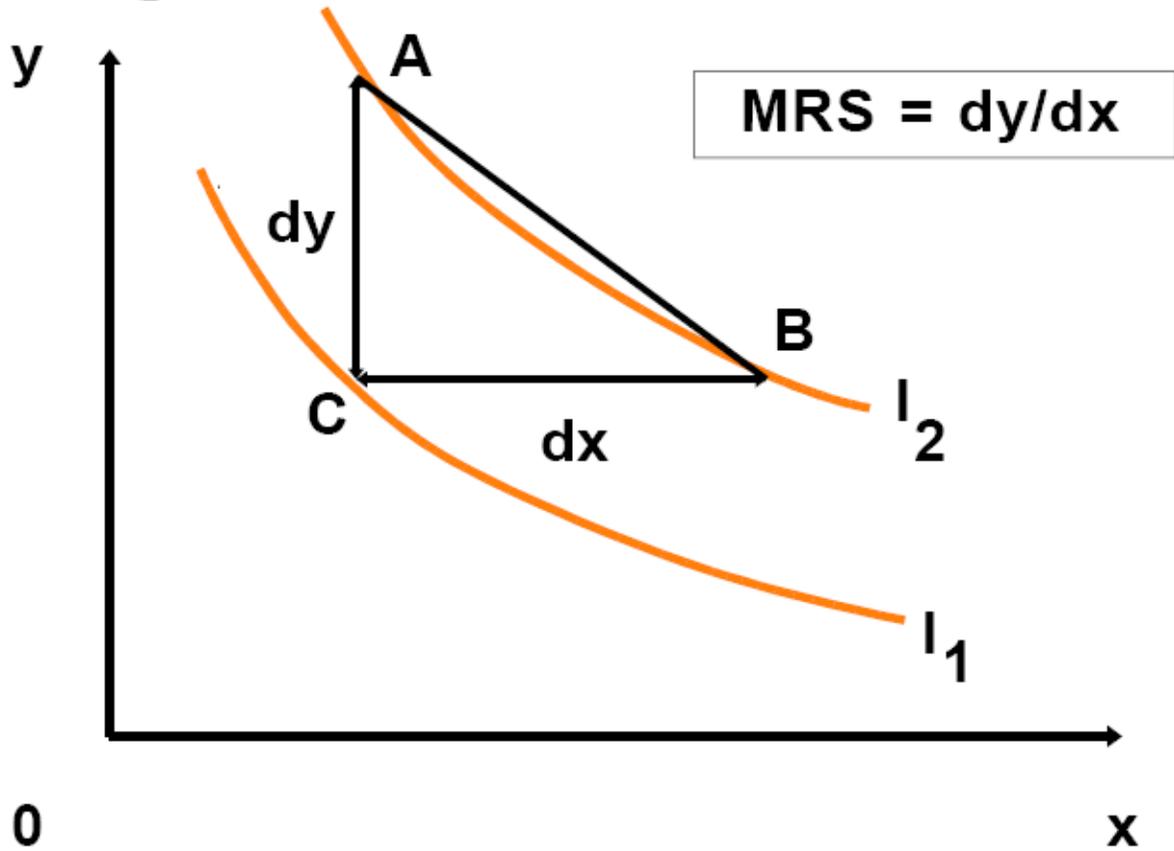
Marginal rate of substitution

$$MRS = dy/dx$$

Figure 6

MOVE	CHANGE IN UTILITY
A to C	$(MU_y)(\Delta y)$
C to B	$(MU_x)(\Delta x)$
TOTAL, A to B	$(MU_x)(\Delta x) + (MU_y)(\Delta y) = 0$

Total change in utility between A and B is zero because A and B are on the

same indifference curve. Rearranging the terms in this we derive the slope of the indifference curve,

$$(MU_y)(\Delta y) = -(MU_x)(\Delta x) \iff MRS = \Delta y/\Delta x = - MU_x/MU_y.$$

12. For a convex indifference curve, its slope goes from high on the left to low on the right. This means that, as the consumer has more Y, she is willing to give up less and less in X in exchange for acquiring equal amounts of Y. Her indifference curves exhibit **diminishing marginal rate of substitution.**

STEP 3: We now find the optimal choice of the consumer by combining the analysis of her preferences with her available choices.

13. Given convex and smooth indifference curves, the consumer maximizes utility at a point A, where the slope of the indifference curve (MRS) is equal to the slope of the budget constraint. At the chosen point A we have *tangency of the indifference curve and the budget constraint line* (Figure 7),

$$\mathbf{p_x/p_y = MRS = MU_x/MU_y,} \ i.e., \ \mathbf{MU_x/p_x = MU_y/p_y.}$$

This means that the consumer receives equal satisfaction for the last dollar spent in each good. The quantity of X that consumer Z chooses at A is called his **demand for X**. The demand of consumer Z varies as prices and income change. We denote it with $x^*(p_x, p_y, I)$. Similarly the demand of this consumer

for good Y is $y^*(p_x, p_y, I)$.

Utility maximization

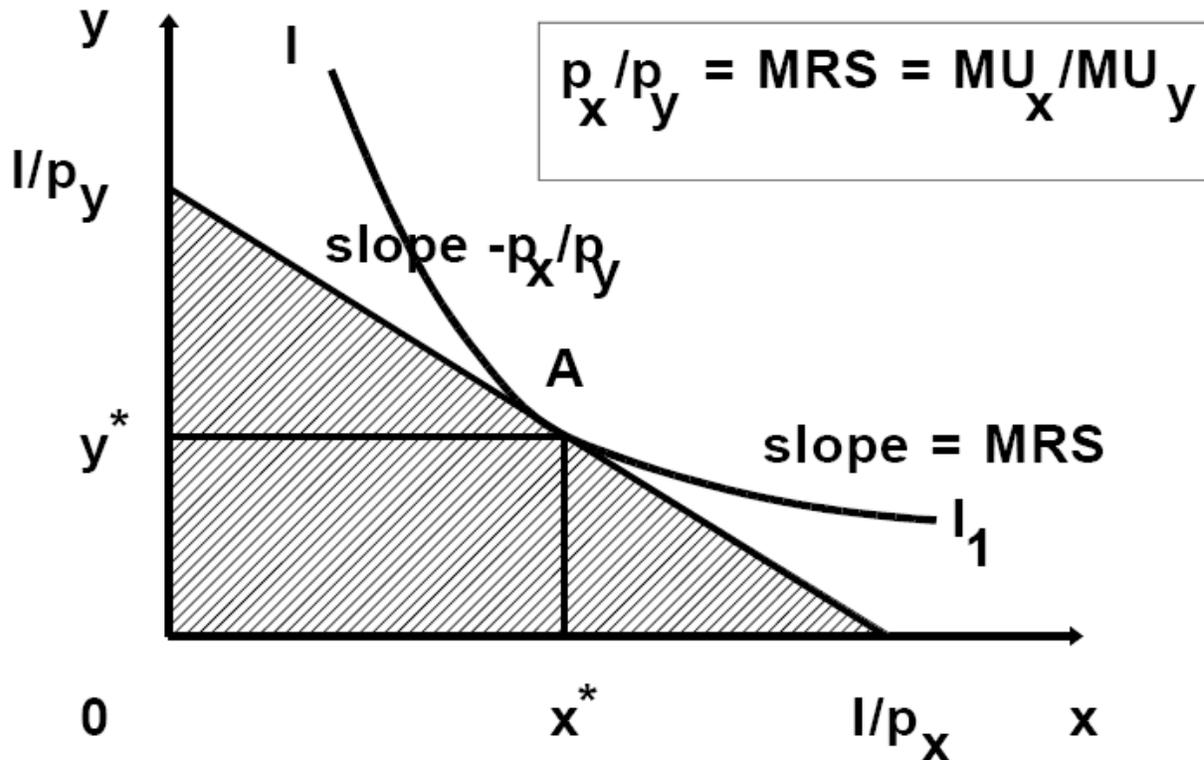$$p_x/p_y = MRS = MU_x/MU_y$$

Figure 7

14. Tangency can **fail** at the optimal point

(1) If the indifference curves are not smooth, for example if they have a kink, as in the case of perfect complements (Figure 8);

(2) If the optimal point is at a corner of the budget set (Figure 9).
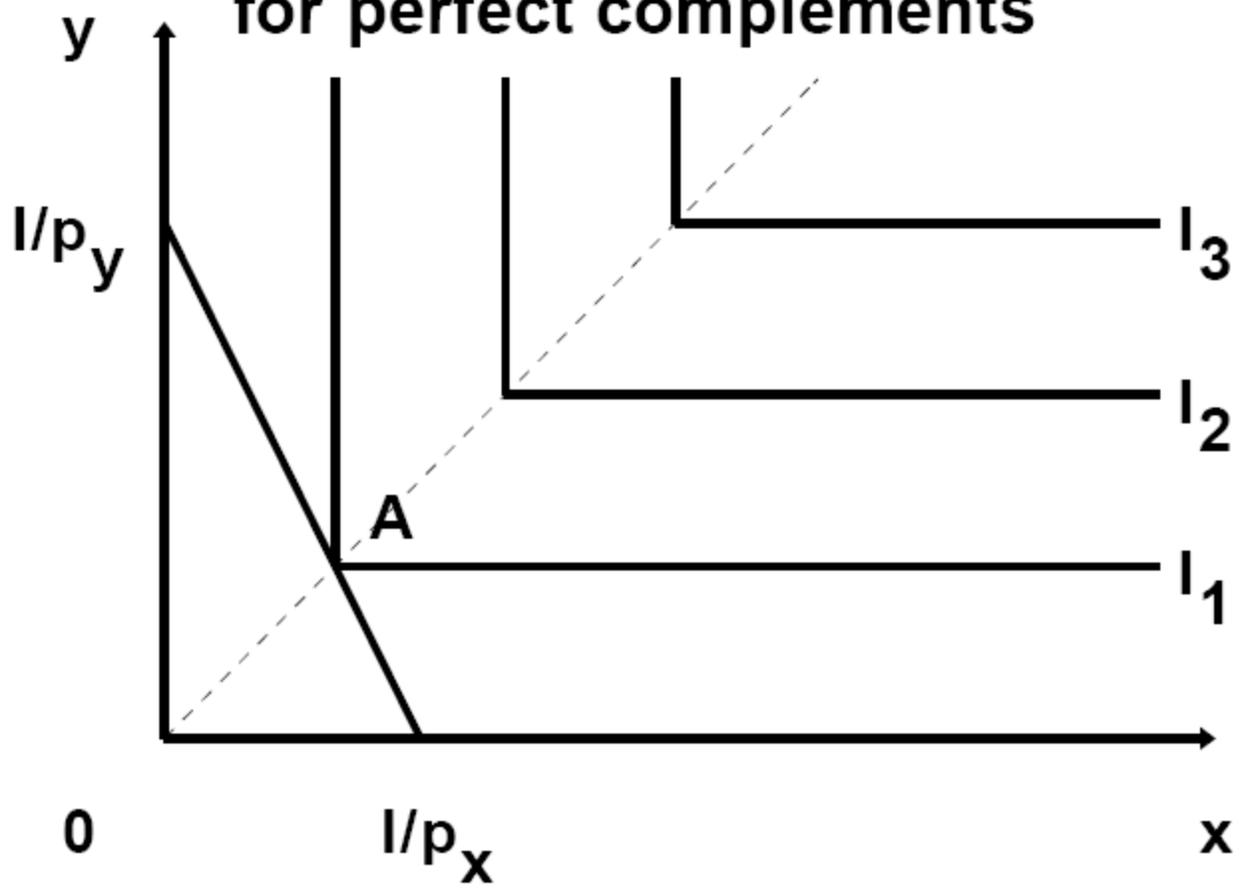
Utility maximization
for perfect complements

Figure 8

Utility maximization
with perfect substitutes

indifference curve

y
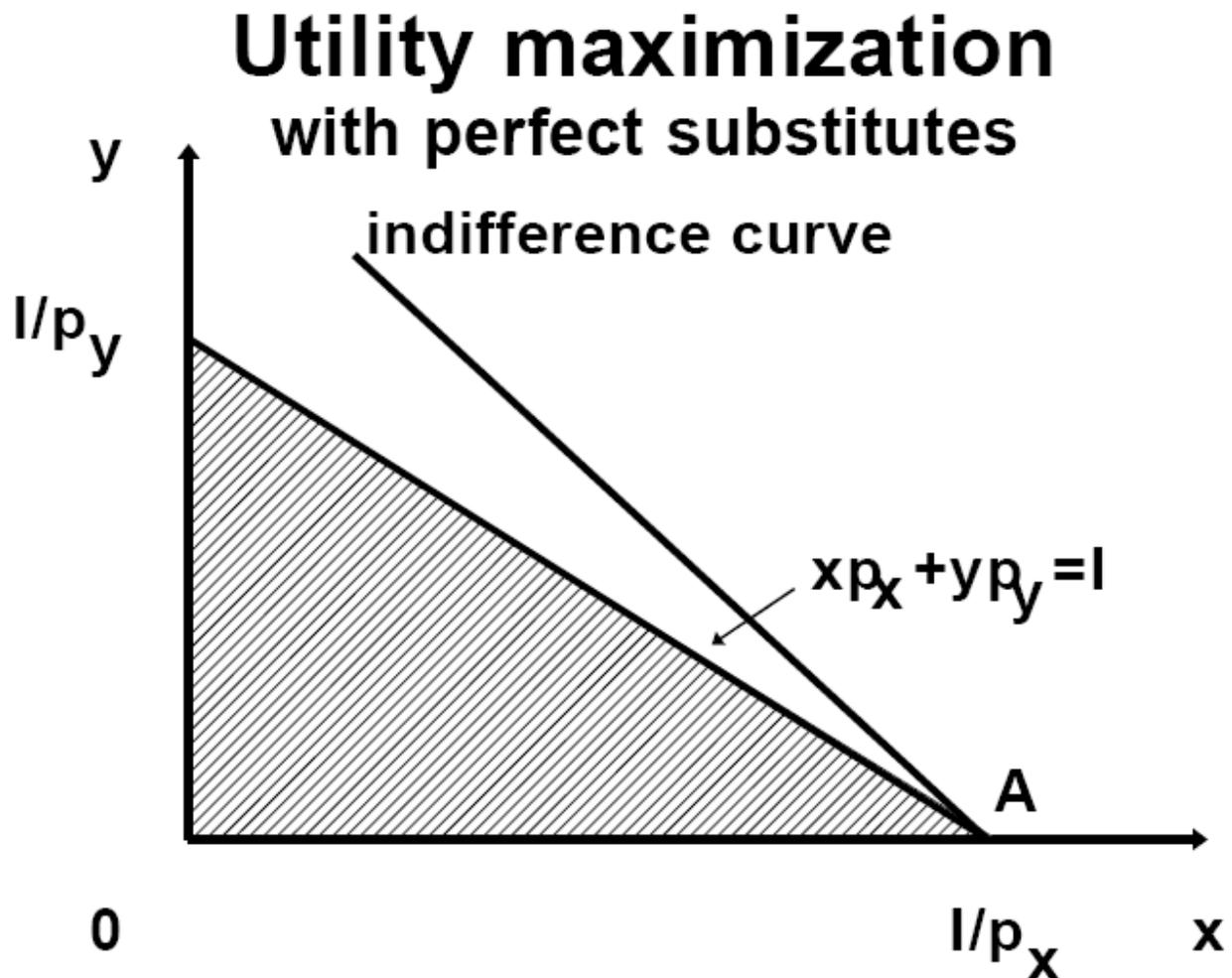
I/p_y

$xp_x + yp_y = I$

A

0

I/p_x

x

Figure 9

15. *Changes in income.* As *income* **expands** the consumer changes his levels

of consumption. If more of X is consumed, then X is a **normal** good (Figure

10). Example: high quality clothes. If less of X is consumed, then X is an

inferior good (Figure 11). Example: low quality food, subway tokens. The

consumption bundles A, B, C, as income increases are on the **income**

expansion path.

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Changes in income

slope - p_x/p_y

Income Expansion Path
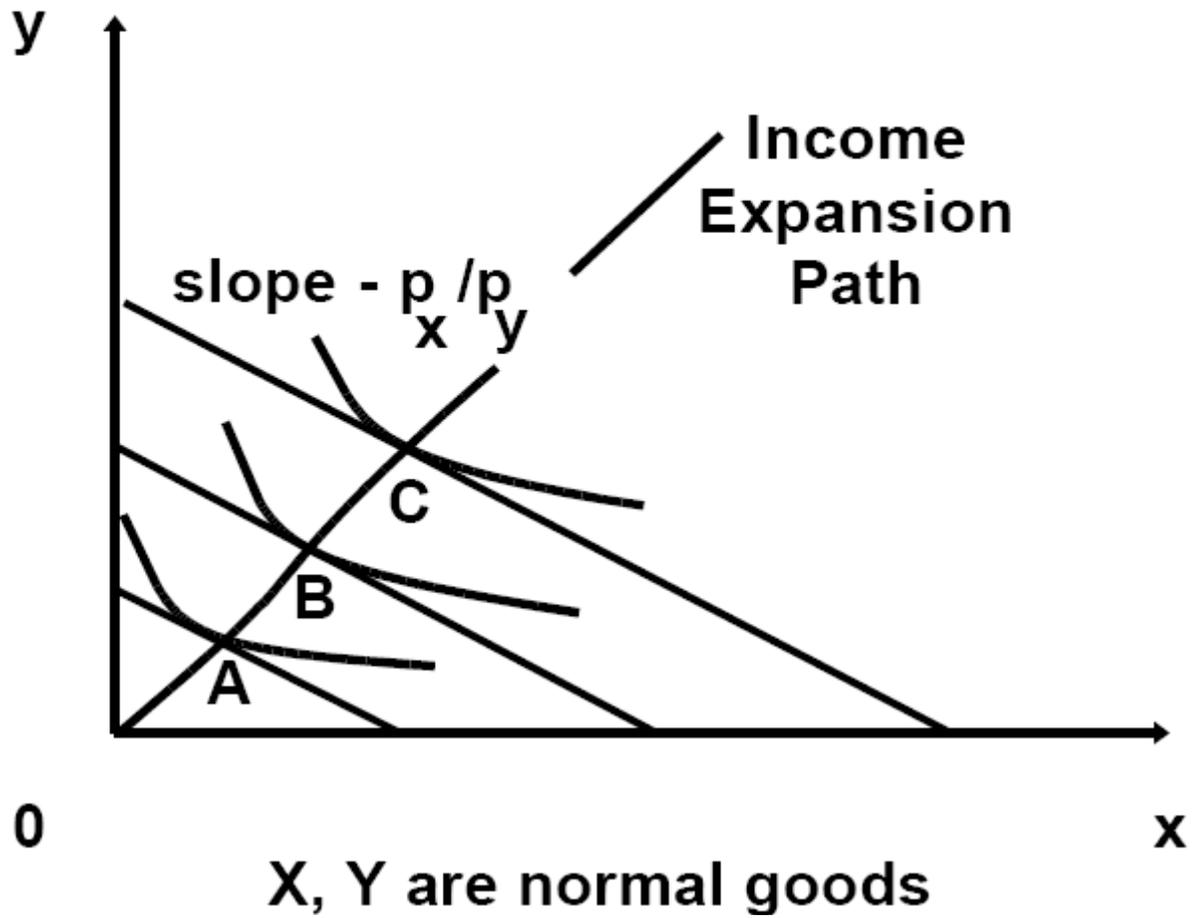
C

B

A

0

X, Y are normal goods

Figure 10
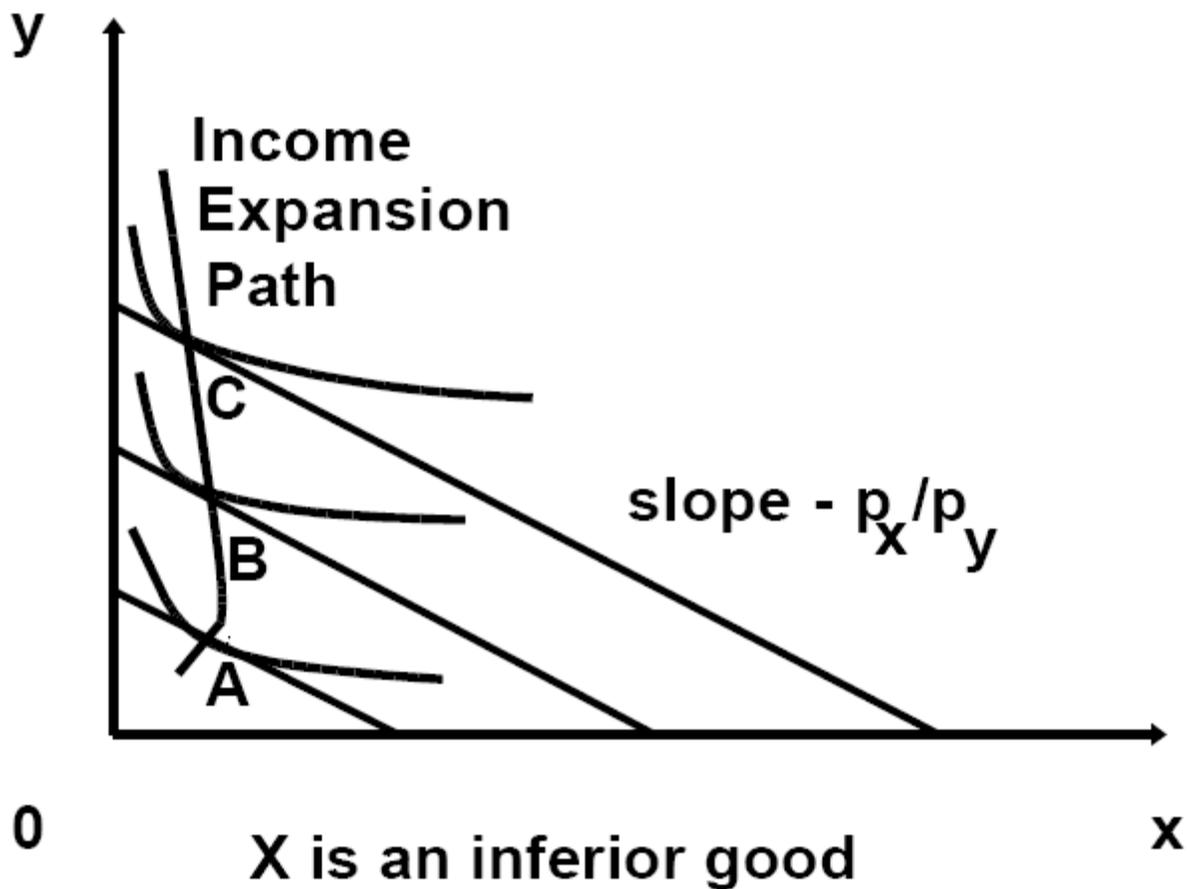
18

Changes in income

Figure 11

16. *Changes in prices.* As the price of X decreases, there is a natural tendency

to consume more of the good that became cheaper. But, at the same time,

because of the price decrease, the consumer suddenly finds herself more

wealthy. She can buy the old bundle, and still have left-over money. We can

separate these two effects on the consumption of X as the **substitution effect**

and the **income effect**. There is a natural tendency to buy more of the cheaper

good. This is measured, roughly speaking, by the **substitution effect**. The

extra money left-over unspent after the price decrease may be spent on X or on

Y. The increase or decrease in the consumption of X resulting from the

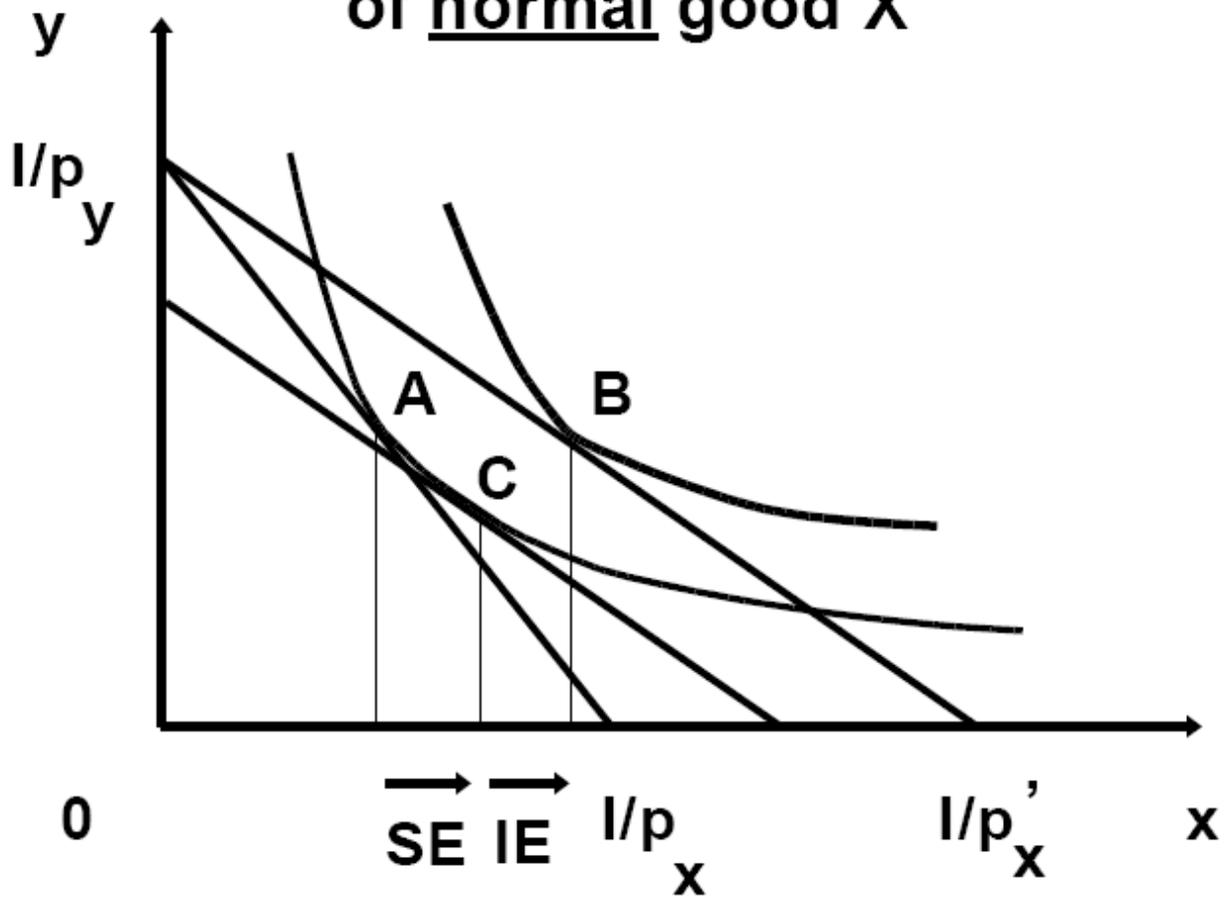# Effects of a price decrease
of <u>normal</u> good X

y

I/p_y

A   B

C

0

$\overrightarrow{SE}$ $\overrightarrow{IE}$ I/p_x I/p_x' x

Figure 12

spending of the left-over money is measured, roughly speaking, by the **income**

effect. Note that a consumer may not like good X as much when she is richer, and could decrease its consumption as her income increases.

17. Consider a price decrease that makes the consumer move from A to B. We break it into the change from A to C (substitution effect) and the change from C to B (income effect). We isolate the substitution effect by taking away from the consumer enough money to put her at the same level of satisfaction as before the price change. In Figure 12, the line through C has the same slope as the one through B, and is tangent to the indifference curve through A.

18. *The direction of the substitution effect is always opposite to the price change*. If the good is **normal**, the income effect is in the same direction. The two effects reinforce each other. Therefore, ***if a good is normal, the demand curve (that shows how much would be sold at different prices) as slopes downward***. This is called **the law of demand**. A typical demand curve is shown in Figure 12a. This far we have used the notation x for quantity and p_x for price. For most of the remaining of the course we will use the notation Q for quantity and P for price. Notice that there are two alternative ways to interpret what a demand curve shows. First, the demand curve shows how many units people are willing to buy at any particular price: Q(P).   Second the

demand curve shows what price would be fetched if a certain number of units

of output were offered at the market: P(Q).
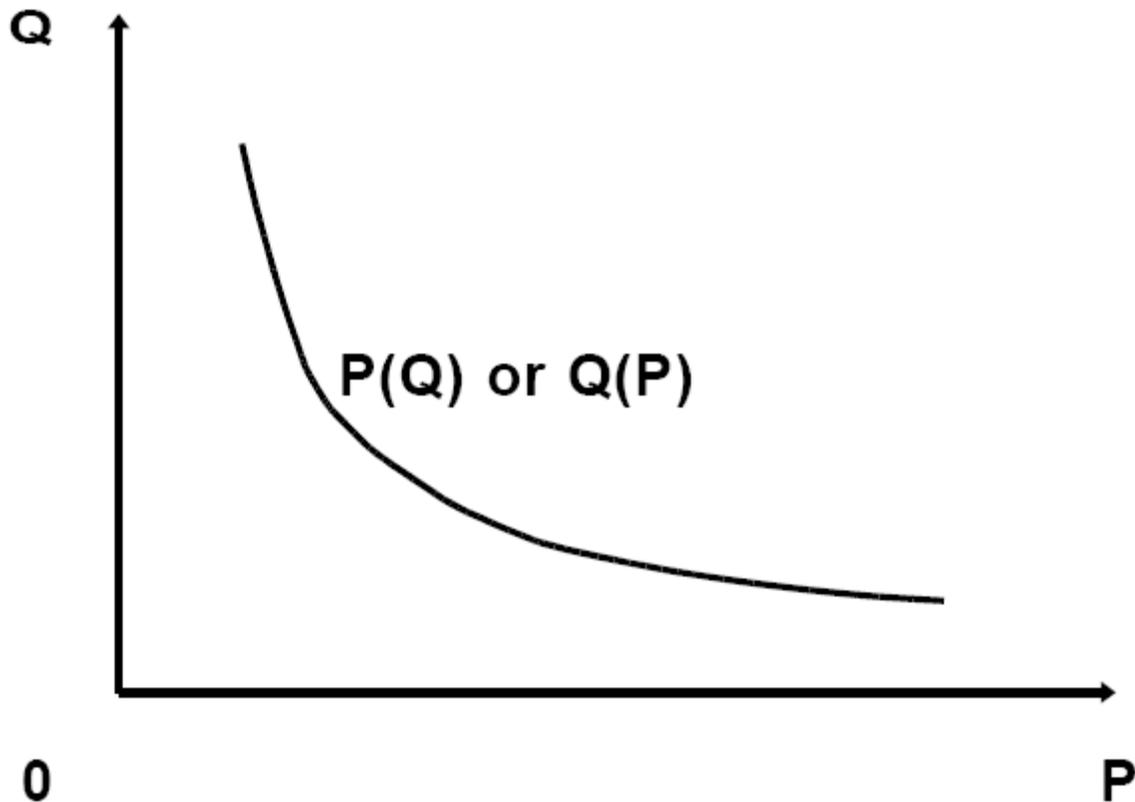
Demand curve

Figure 12a

19. If the good is **inferior**, the income effect goes in the opposite direction of

the substitution effect (Figure 13). *Typically, even for inferior goods, the*

demand slopes downward, because the income effect is smaller than the

substitution effect. If the good is very strongly inferior (in very rare cases), the

income effect could be bigger than the substitution effect, so that the total effect is opposite to the substitution effect, and the demand slopes upward. Then the good is called a **Giffen good**.

Effects of a price decrease
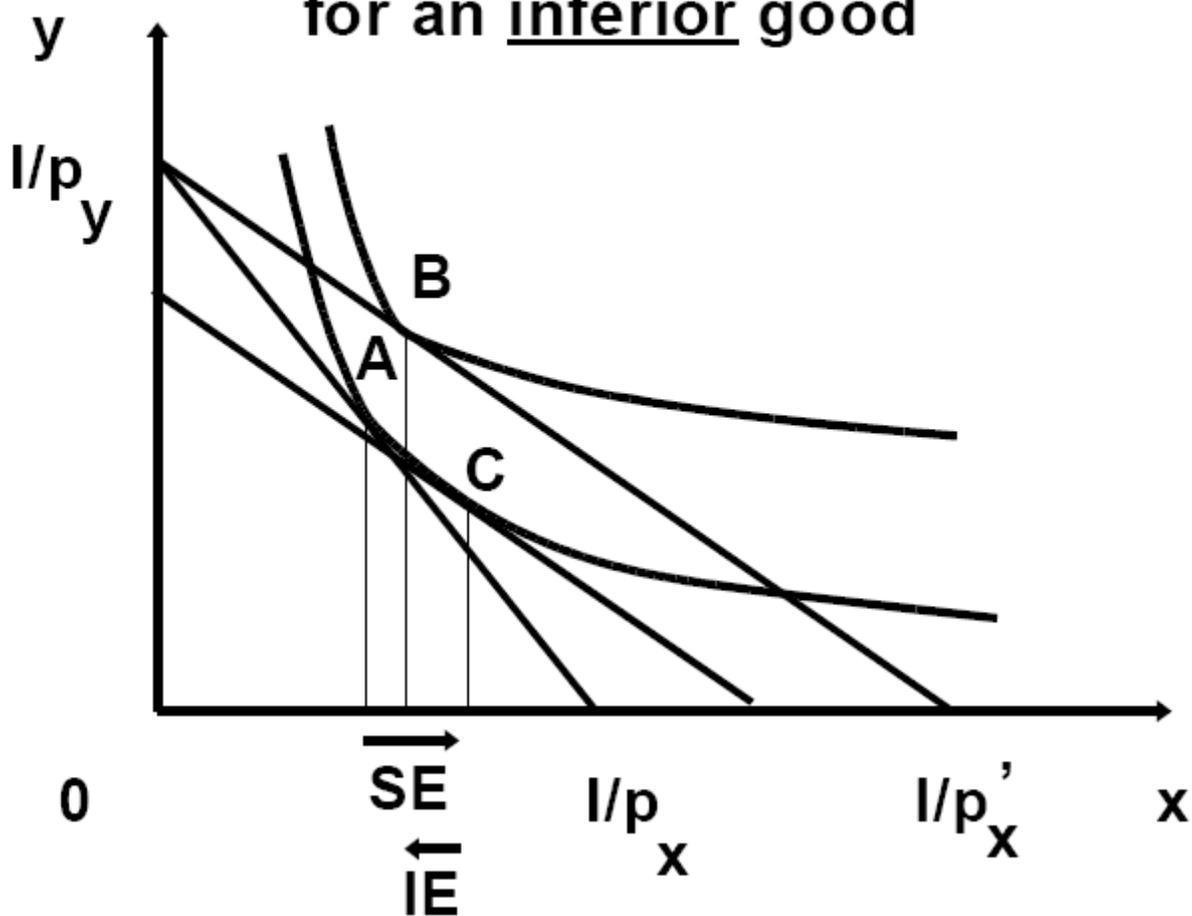## for an <u>inferior</u> good

Figure 13

20. **Specific taxes and taxes on wealth** (Figure 14). The income-substitution analysis can be applied to taxation. Consider two alternative taxation schemes.

In the first, only good X (say gasoline) is taxed, so that its price goes from p_x to $p_x + t$, where t is the tax per gallon. In the second wealth, I, is taxed. Suppose that both schemes *raise the same amount of total tax*. Then the second scheme leaves consumers better off (*i.e.*, on a higher indifference curve).
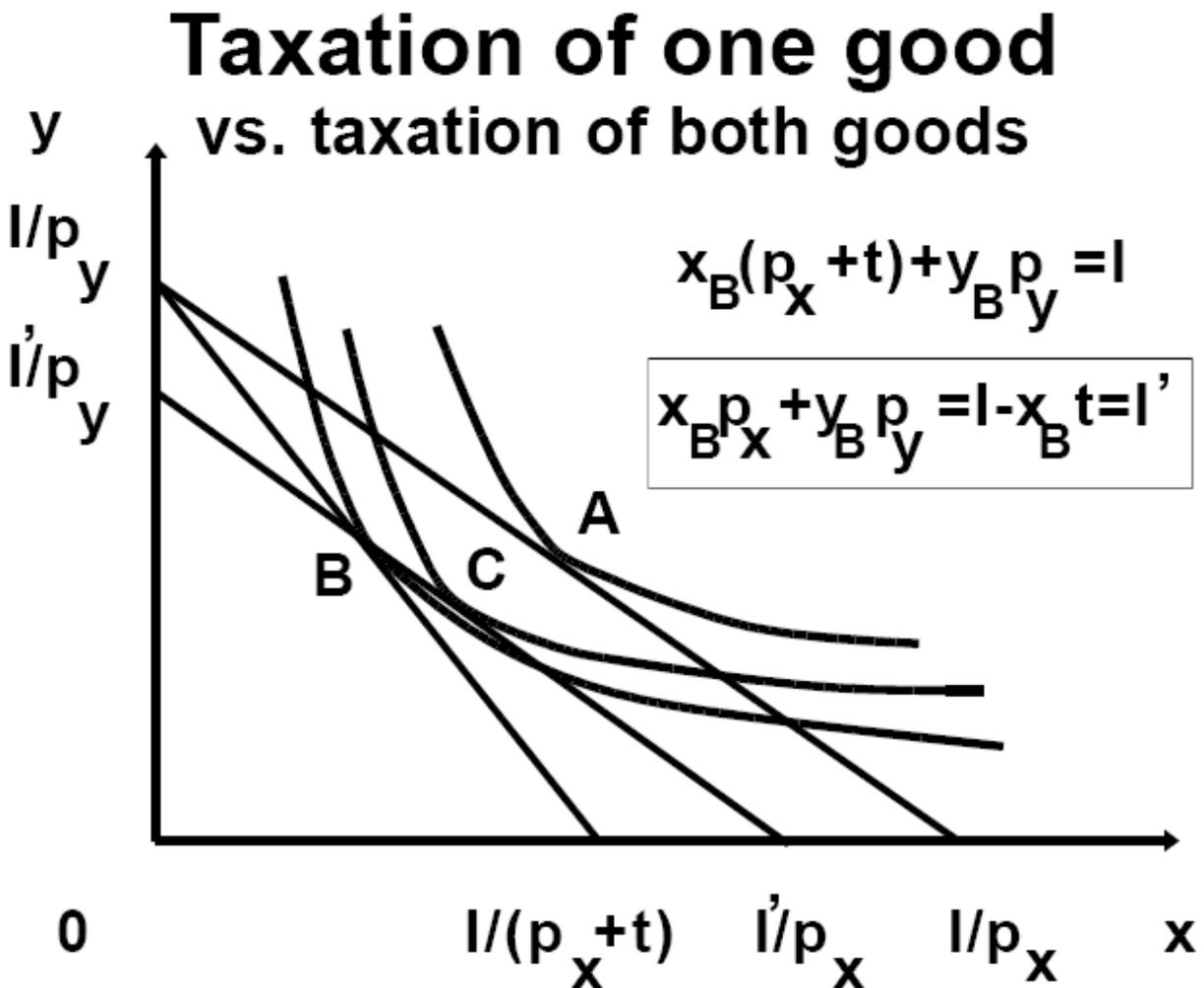
Taxation of one good vs. taxation of both goods

$$x_B(p_x + t) + y_B p_y = I$$

$$x_B p_x + y_B p_y = I - x_B t = I'$$

Figure 14

21. *Elasticities measure the responsiveness of quantities traded to prices or*

income. **Price elasticity** measures the **percentage change** in quantity as a response to a **percentage change** in price. The (own) **price elasticity of demand** is

$$e = (\Delta Q/Q)/(\Delta p/p) = (\Delta Q/\Delta p)(p/q).$$

Note that $e < 0$, and that the elasticity is <u>not</u> the slope of the demand curve. For example, for a linear demand curve $Q = a - bP$, the slope is $\Delta Q/\Delta p = -b$ (constant) but the elasticity of demand is $-bp/q$ which varies as the quantity (or price) changes.

If $e < -1$, *i.e.*, $|e| > 1$, the demand is **elastic**, *i.e.*, highly responsive to changes in price. Typical for non-necessities, goods you do not have to buy, luxuries.

If $e > -1$, *i.e.*, $|e| < 1$, the demand is **inelastic**, *i.e.*, not responsive to changes in price. Typical for necessities, goods that you have to buy.

If $e = -1$, *i.e.*, $|e| = 1$, the demand is called **uni-elastic**.

Generally, a demand is more elastic if the product has close substitutes. Demand for an individual brand is more elastic than market demand. Typically, elasticity of long run demand is higher than elasticity of short run demand because of wider availability of substitutes.

22. The **income elasticity of demand** measures the responsiveness of the

quantity demanded on income changes.

$$e_I = (\Delta Q/Q)/(\Delta I/I) = (\Delta Q/\Delta I)(I/Q).$$

If $e_I > 0$, the good is **normal**. If $e_I < 0$, the good is **inferior**. Most goods are

normal. However, goods you buy when you have low income may be inferior.

23. The **cross elasticity of demand** measures the responsiveness of the

demand for good X on price changes of another good, Y.

$$e_{x,py} = (\Delta x/x)/(\Delta p_y/p_y).$$

If $e_{x,py} > 0$, x and y are **substitutes** (say Diet Coke and Diet Pepsi). If $e_{x,py} < 0$, x

and y are **complements** (say computers and printers).

24. If all units are sold *at the same price,* the consumers who are willing to buy

at a high price benefit from the existence of consumers who are willing to pay

only a low price. All units are sold at a price equal to the willingness to pay for

the last unit. The difference between what a consumer is willing to pay and

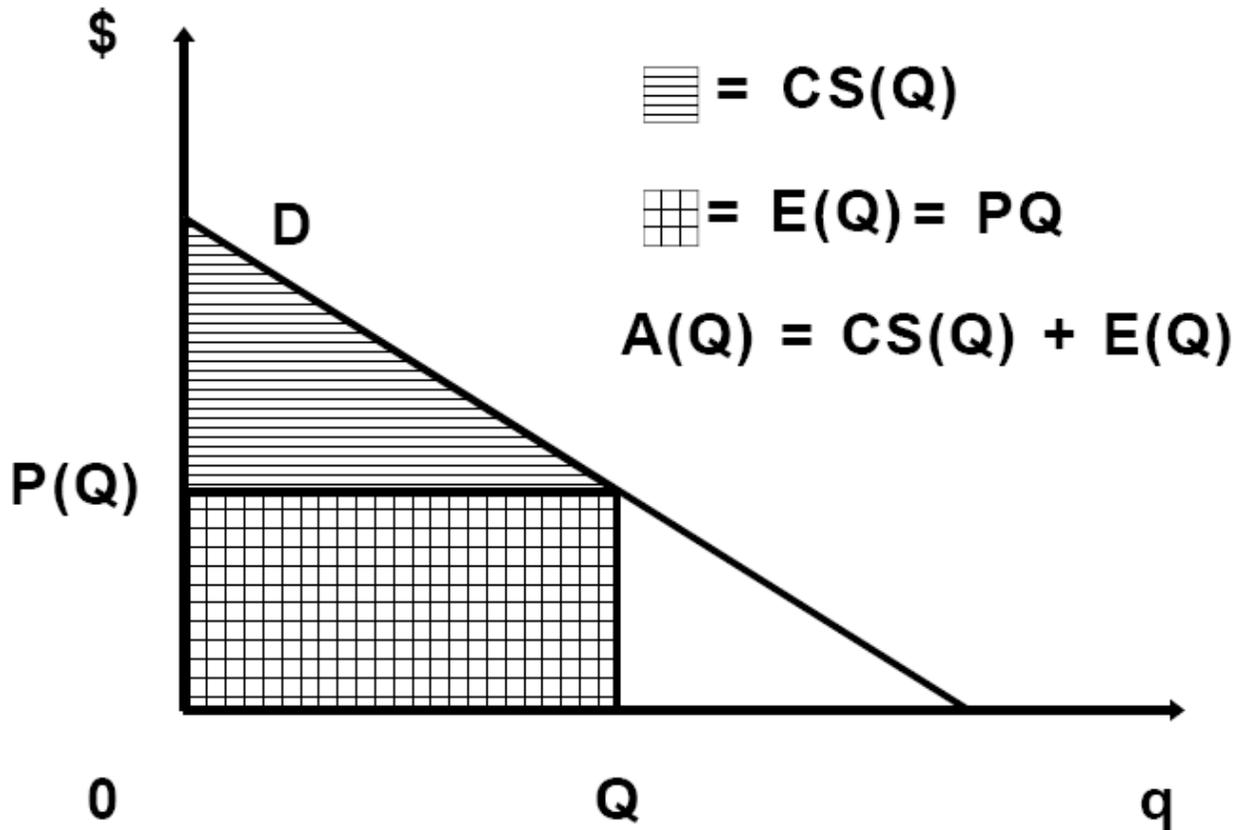what he actually pays is called **consumers surplus**.

Consumers' surplus

Figure 15

25. The total willingness to pay up to Q units is the area under the demand up to Q units, A(Q). The actual expenditure is E(Q) = QP(Q). The difference is **consumers' surplus**,

$$\text{CS(Q)} = \text{A(Q)} - \text{E(Q)}.$$

In Figure 15, expenditure E(Q) is double-shaded, and consumers' surplus CS(Q) is single-shaded. A(Q), the total willingness covers both shaded areas.

B.   CONSUMPTION ANALYSIS UNDER UNCERTAINTY

26. **Risk and Uncertainty**.

Example 1: Consider the choice between receiving $10 with certainty and

receiving $8 or $12 with probability 1/2 each. Note that both alternatives have

the same expected value of $10. The utility of the first alternative is U(10), and

the utility of the second alternative is U(8)/2 + U(12)/2. Remember, both

alternatives have the same expected monetary value of $10, but only the first

one guarantees this amount with certainty. A **risk-averse** person will prefer the

first (riskless) alternative. This means that for a risk averse person,

$$U(10) > U(8)/2 + U(12)/2.$$

Note that, for a risk-averse person, the utility of wealth is concave. See Figure

16. This means that the marginal utility of wealth (the utility of the last dollar)
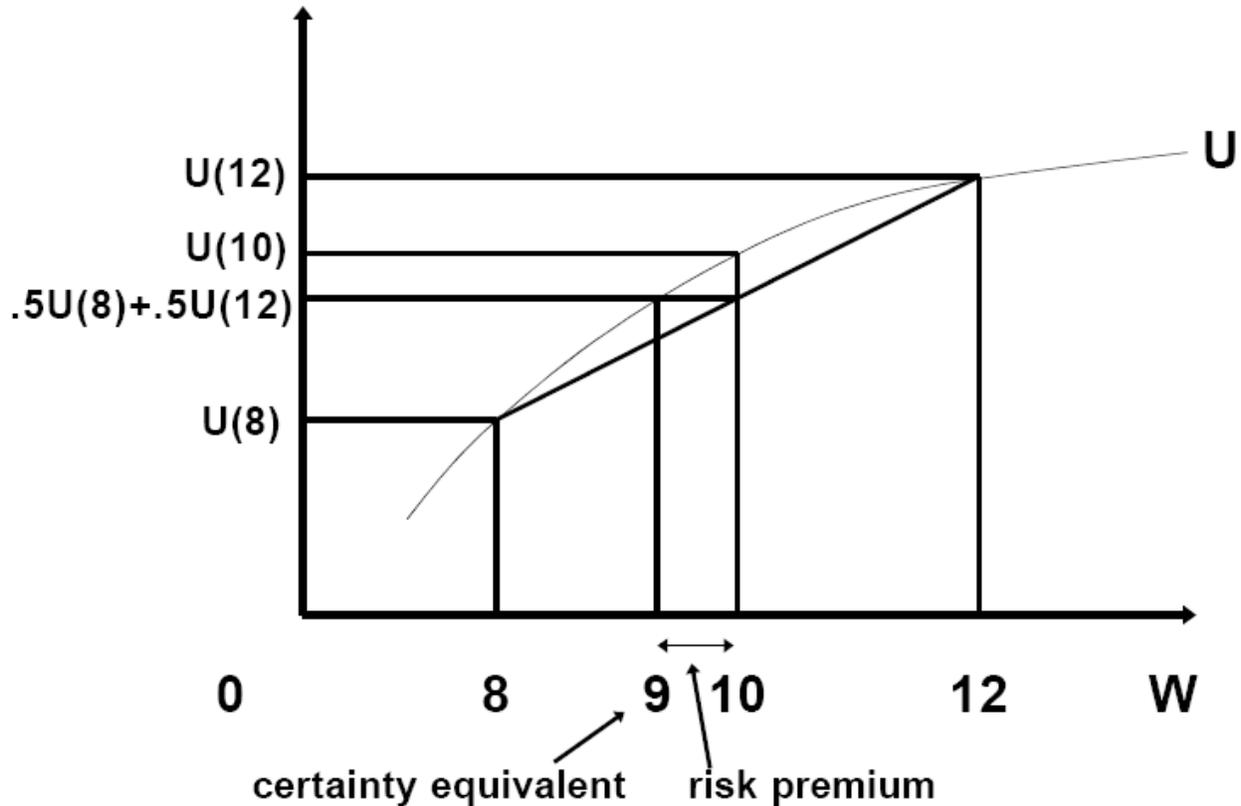
is decreasing with wealth.

Risk Averse Consumer

Figure 16

A **risk-lover** will prefer the second (risky) alternative, *i.e.,* for him

$$U(10) < U(8)/2 + U(12)/2.$$

Note that, for a risk-lover, the utility of wealth is convex. See Figure 17. This means that the marginal utility of wealth (the utility of the last dollar) is increasing with wealth.
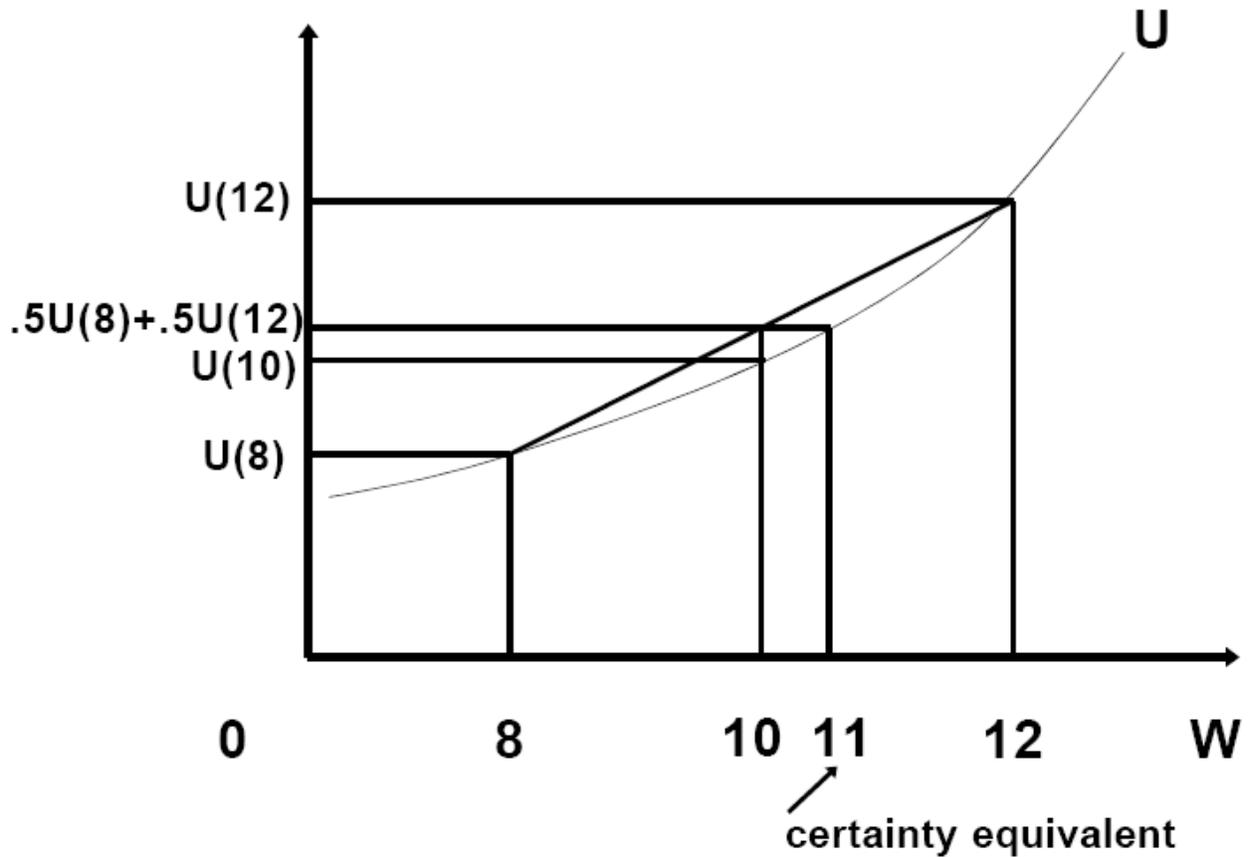
Risk Loving Consumer

Figure 17

A **risk-neutral** person is indifferent between the two alternatives,

$$U(10) = U(8)/2 + U(12)/2.$$

For a risk-neutral person, the utility of wealth is a straight line. This means that the marginal utility of wealth (the utility of the last dollar) is constant for any level of wealth.

*We define the **certainty-equivalent** of an uncertain situation as the amount of*

money x that, if received with certainty, is considered equally desirable as the uncertain situation, i.e.,

$$U(x) = U(8)/2 + U(12)/2.$$

For a risk-averse person the certainty equivalent must be less than 10, $x < 10$.

For a risk-neutral person $x = 10$, and for a risk-lover $x > 10$.

Example 2: Suppose that a consumer has utility function

$$U(W) = \sqrt{W} = W^{1/2},$$

where W is her wealth. First check that the consumer is risk averse (either by showing that $\sqrt{W}$ is concave or by showing that he prefers \$10 rather than the \$8 or \$12 lottery above). Suppose that this consumer with probability 1/200 (= 5/1000) has \$10,000, and with probability 99/200 (= 995/1000) has \$1,000,000. How much will she be willing to accept with certainty in return for this uncertain situation?

Solution: Facing the uncertain situation, her utility is

$.995*\sqrt{1,000,000} + .005*\sqrt{10,000} = .995*1,000 + .005*100 = 995+0.5 = 995.5$

The **certainty-equivalent** is \$x such that $\sqrt{x} = 995.5$, *i.e.*, $x = \$(995.5)^2 =$ \$991,020.25. The consumer is willing to accept \$991,020.25 with certainty in return for her uncertain situation. Note that because the consumer is risk averse

(equivalently because $\sqrt{W}$ is concave) the certainty-equivalent x = $991,020.25

is smaller than the expected dollar value of the uncertain situation which is

$$.995*1,000,000 + .005*10,000 = 995,000 + 50 = \$995,050.$$

27. **Insurance**. Suppose that a consumer has wealth W_G with probability $prob_G$

(if no "bad" event occurs), but his wealth is diminished to W_B with probability

$prob_B$ (if the "bad" event occurs). Since these are the only two eventualities,

$prob_G + prob_B = 1$ must be true. The bad event creates a loss of $L = W_G - W_B$.

In general, an insurance company is willing to take away an uncertain situation

and give in return to the consumer the same amount of money in both the

"good" and "bad" eventualities. To be able to guarantee the same wealth even

if the bad event happens, the insurance company collects a fee, called the

premium. Thus, the insurance contract takes away from the consumer the

uncertain world $\{W_G$ with probability $prob_G$, W_B with probability $prob_B\}$ and

replaces it with a certainty world $\{W, W\}$. The remaining question is how does

W relate to W_G and W_B. To start with, it must be that W is between the high

and low wealth $W_G < W < W_B$ (why?), but we need to find more specifically

what W is acceptable to the consumer.

Example 3: A consumer with utility function $U(W) = \sqrt{W}$ has wealth from two

sources. He has $10,000 in cash, and a house worth $990,000. Suppose that the house burns down completely with probability 0.005 (that is 0.5% or 5 in a thousand). The house lot is assumed to be of no value. How much is the consumer willing to pay for full insurance coverage?

Notice that the consumer has wealth $1,000,000 with probability 0.995, and $10,000 with probability 0.005, so her uncertain situation is exactly as in example 2. We have found above that the certainty equivalent of this uncertain situation for this consumer is $991,020.25. Therefore she is willing to accept any insurance contract that would leave her with wealth of at least $991,020.25 (hopefully higher). The insurance contract that is just acceptable is the one that leaves her with exactly $991,020.25. In insurance terms, this is a contract with coverage of $990,000 and premium P_{max}. To find the maximum premium the consumer is willing to pay, P_{max}, we reason as follows. If the house does not burn down, the consumer has wealth of $991,020.25 = $1,000,000 (original wealth) – P_{max} (premium). If the house burns down, the consumer has again wealth $991,020.25 = $10,000 (cash) + $990,000 (payment from the insurance company) – P_{max} (premium). Therefore the maximum premium acceptable to the consumer is P_{max} = $1,000,000 - $991,020.25 = $8,979.75.

Of course, the consumer is willing to accept full coverage of $990,000 for a lower premium than P_{max}. One such premium is the one that corresponds to an **actuarially fair** insurance contract that guarantees with certainty the dollar expected value of the wealth of the uncertain situation it replaces. (In example 1 this expected value is $10.) In example 2, the expected value of the wealth of the uncertain situation is (as we have calculated above)

$$.995*1,000,000 + .005*10,000 = 995,000 + 50 = \$995,050.$$

Therefore the actuarially fair premium is $P_{act.fair} = \$1,000,000 - \$995,050 = \$4,950$. If the insurance company is risk neutral, charging the actuarially fair premium allows the company to just break even. Any additional premium above the actuarially fair is profit to the risk neutral insurance company. Therefore the insurance company can have a profit of up to $P_{max} - P_{act.fair} = \$8979.75 - \$4,950 = \$4029.75$. In summary, a risk neutral insurance company will not charge a premium below $4,950 and the consumer will not accept a premium above $8,979.75. The premium will be between these two numbers and will depend on competition among insurance companies.

C. PRODUCTION AND COSTS

28. *Output* Q is produced from many *inputs* including labor man-hours L,

capital (machine hours) K, land use, and other factors. For the purposes of this course, we will assume a single variable factor of production, labor L. The relationship between labor and output is summarized by the *production function* $Q = f(L)$. We define the *marginal productivity* of labor as $MP_L = df/dL$. We also expect that marginal productivity MP_L to be *decreasing* at high usage levels of the particular input (L). See Figure 18. We also define the *average productivity*

$$AP_L = Q/L = f(L)/L.$$

Since marginal productivity eventually (*i.e.*, with high use of the input) decreases, it will eventually drive the average productivity down too.

29. The **cost** of production of Q units is the cost $wL of the input L required to produce quantity $Q = f(L)$, where $w is the price of labor. For example if $Q = f(L) = 10L$, then the amount L required to produce Q units is Q/10, and therefore the cost to produce Q is $C(Q) = wQ/10$. Similarly, if $Q = f(L) = 10\sqrt{L}$, the amount L required to produce Q units is $(Q/10)^2$ and therefore the cost to produce Q is $C(Q) = w(Q/10)^2$.
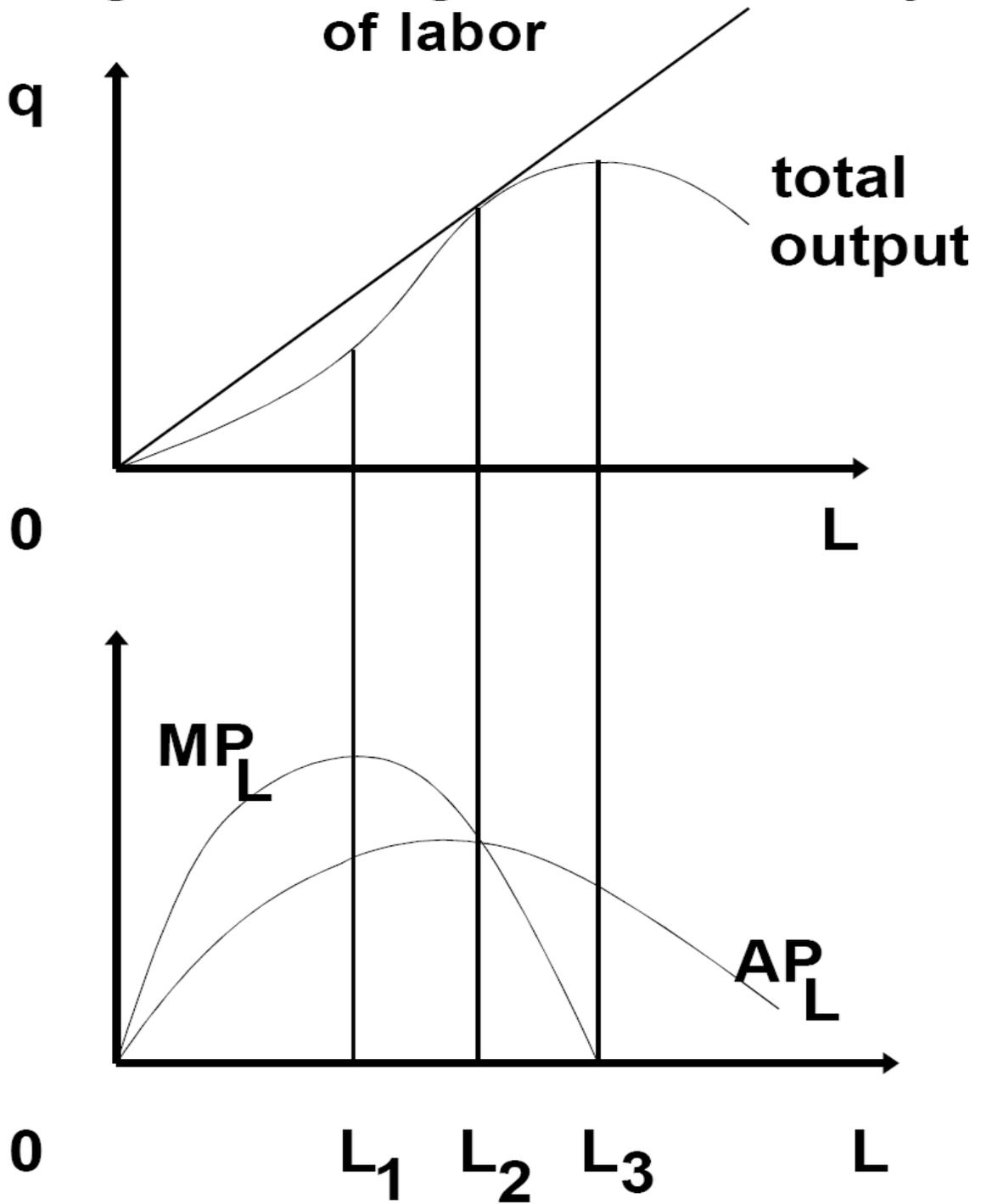
Average and Marginal Productivity of labor

Figure 18

30. **Returns to scale**. A production function exhibits **constant returns to scale** (CRS) if doubling the input results in double output,

$$f(2L) = 2f(L).$$

A production function exhibits **increasing returns to scale** (IRS) if doubling the input results in more than double output,

$$f(2L) > 2fL).$$

A production function exhibits **decreasing returns to scale** (DRS) if doubling the inputs results in less than double output,

$$f(2L) < 2f(L).$$

31. **Total, fixed, variable, average**, and **marginal costs**. Fixed or setup costs do not vary with the level of production.

Total costs: $C(q)$ or $TC(q)$.

Variable costs: $V(q)$.

Fixed costs: F, constant.

Breakdown of total costs $C(q) = F + V(q)$.

Average total cost: $ATC(q) = C(q)/q$.

Average variable cost: $AVC(q) = V(q)/q$.

Average fixed cost: $AFC(q) = F/q$.

$$ATC(q) = F/q + AVC(q).$$

Incremental (marginal) cost: $MC(q) = C'(q) = dC/dq = V'(q) = dV/dq.$

Incremental or marginal cost is the cost of production of an extra unit of q. Since extra production does not affect fixed cost, the increase in the total cost is the same as the increase in the variable cost, $MC(q) = dC/dq = dV/dq.$

32. We will examine three types of cost functions. The first type of cost functions exhibits constant returns to scale. Every unit of output costs the same to produce. Marginal cost is equal to average cost, $AC(q) = MC(q).$ See Figure 19. Goods with constant returns to scale are typically hand-made where the same amount of labor needs to be put in for every unit.

33. The second type of cost functions exhibits increasing returns to scale for every level of production. Marginal cost is the same for every unit, but there is an additional fixed or setup cost. See Figure 20. Many manufactured goods require a fixed cost for the creation of a design and thereafter the incremental costs are constant. Examples: software, microchips.

34. The third type of cost functions exhibits increasing returns to scale for small production levels and decreasing returns to scale for large production levels.   See Figure 21. Example: traditional manufacturing.

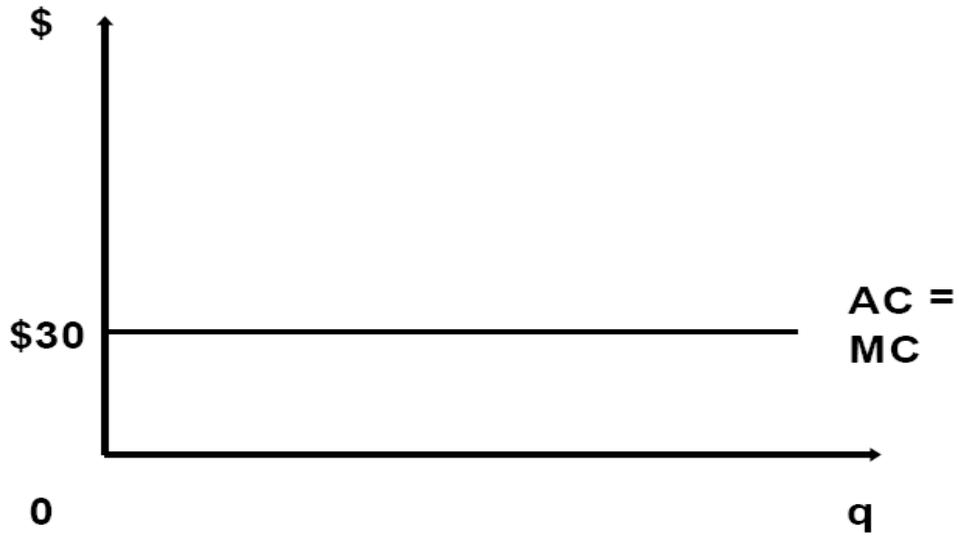# Unit Costs for
Constant Returns to Scale

Figure 19

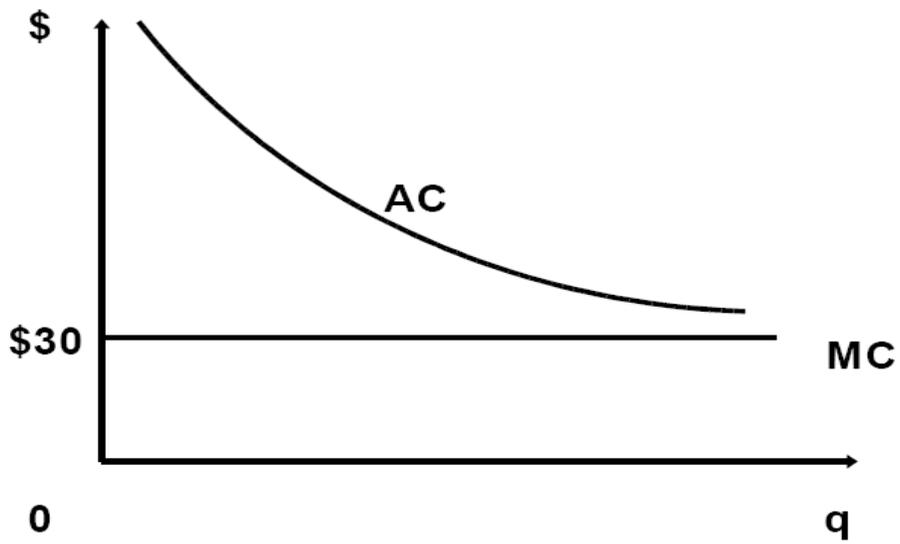# Unit Costs for
Increasing Returns to Scale

Figure 20

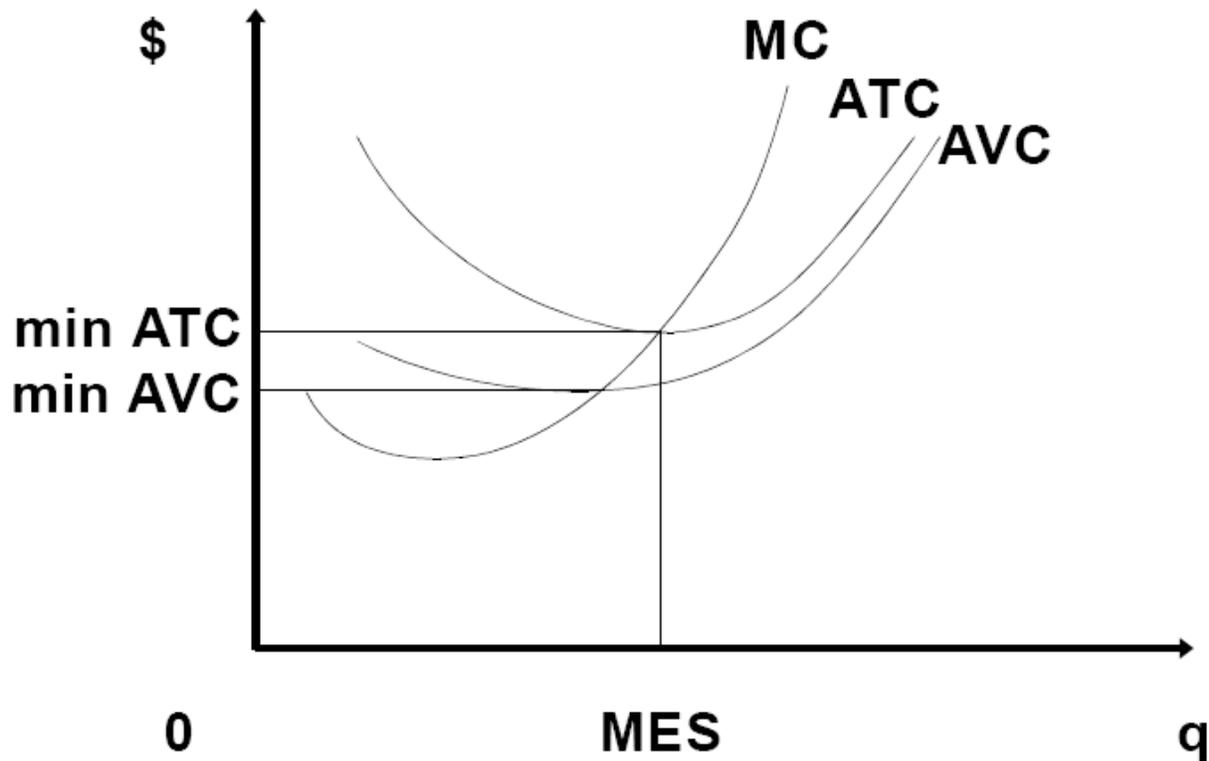# Average and Marginal Costs, and Minimum Efficient Scale (MES)

Figure 21

35. **Profit maximization**. Profits, $\Pi(q)$, generated by production level q are defined as revenues, R(q), minus costs, C(q),

$$\Pi(q) = R(q) - C(q).$$

In the simplest case, **revenues** are price times quantity $R(q) = pq$.
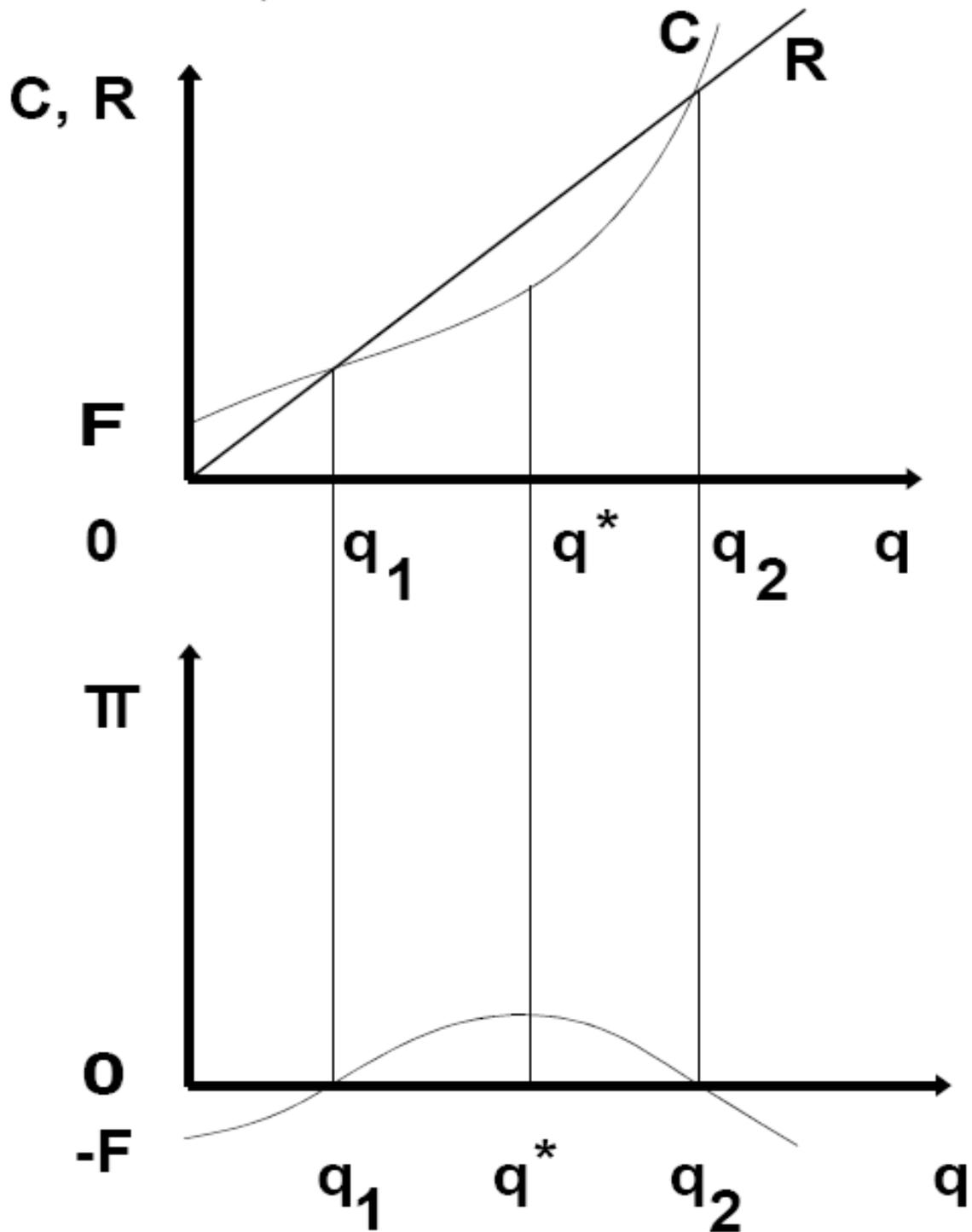
Costs, Revenues and Profits

Figure 23

36. Typically, profits are negative for small q because of positive setup costs that cannot be recovered at low sales. Profits increase in q, reach a maximum, and then decrease. See Figure 23. In this course, we will assume that the firm tries to maximize profits. At the quantity level q^* that maximizes profits, the slope of $\Pi(q)$, $d\Pi/dq$ (incremental or marginal profit), is zero. At any q,

$$d\Pi/dq = R'(q) - C'(q) = MR(q) - MC(q).$$

that is, incremental profit is always equal to incremental revenue minus incremental cost. Therefore, at q^* that maximizes profits, marginal revenue is equal to marginal cost,

$$d\Pi/dq = 0 \quad \textit{i.e.,} \ \ MR(q^*) = MC(q^*).$$

This condition is necessary for profit maximization irrespective of the organization of the industry and of the nature of competition. As we will see below, the nature and extent of competition in a market changes MR but usually has no significant influence on MC.

D.   MARKET STRUCTURE

37. *By definition, in* **perfect competition**, *no firm has any influence on the market price*. Therefore, each unit of output is sold at the same price, p. Therefore, for a competitive firm, $MR(q) = p$. Each firm perceives a horizontal

demand for its output (at price p).

38. A competitive firm chooses q^* so that

$$p = MR = MC(q^*).$$

See Figure 24. If the price falls below minimum average total cost, the firm

closes down. Therefore *the **supply function** of a competitive firm in the long*

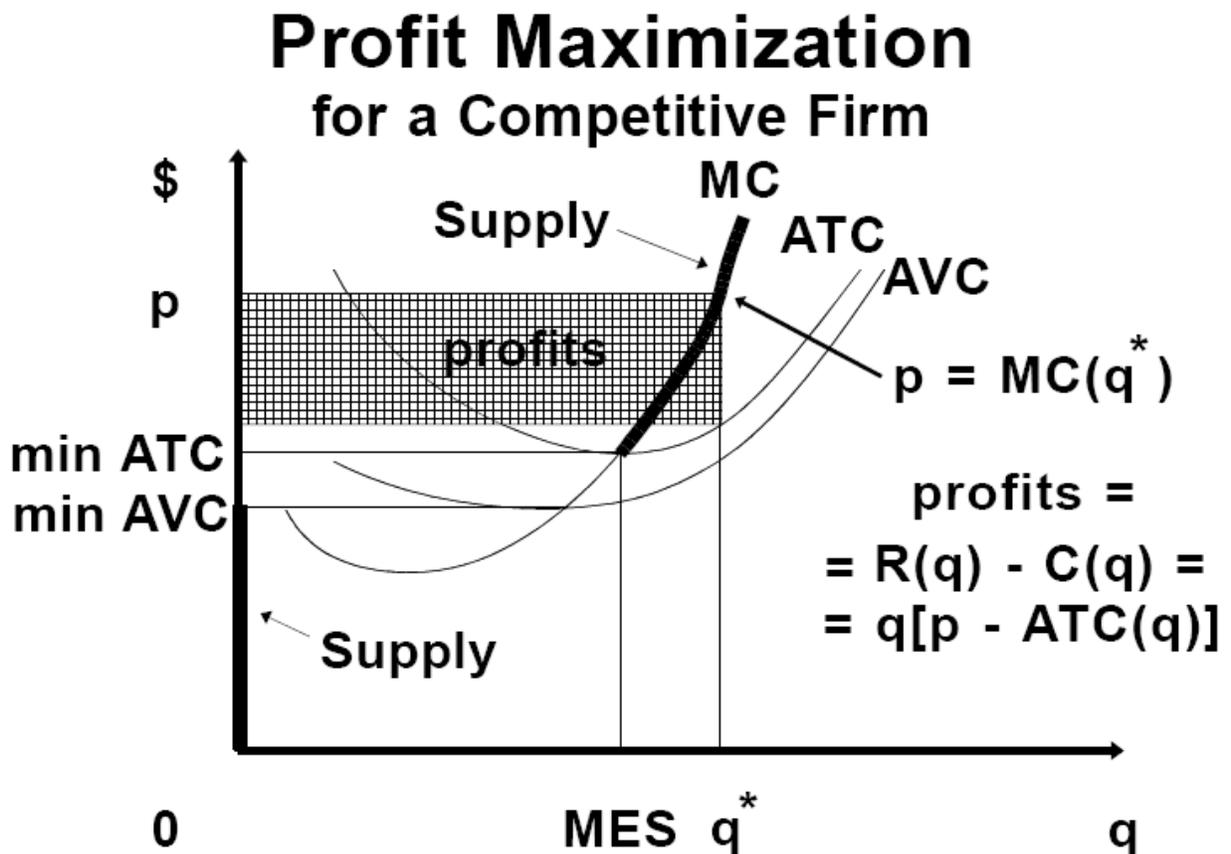*run is its marginal cost curve above minimum ATC.*

Figure 24

For prices below minimum ATC, the firm supplies nothing, q = 0. In the short

run, a firm cannot recover its fixed costs (by closing down). Therefore the

relevant cost function in the short run is average variable cost, AVC. *The firm's*

supply function in the short run is MC above minimum AVC. In the short run, it

supplies zero if p < min AVC. This means that there is a range of market

prices, between min ATC and min AVC, such that the firm will

produce a positive quantity in the short run, but will shut down in the long run.

Profits are the shaded box in Figure 24,

$$\Pi(q^*) = R(q^*) - C(q^*) = q^*[p - AC(q^*)].$$

Figure 25 shows the market supply with free entry of firms.

Market Supply
in Perfect Competition

Figure 25

Producers' surplus

$$S = MC$$

$$\boxed{\hspace{0.3cm}} = V(q)$$

$$\boxed{\hspace{0.3cm}} + \boxed{\hspace{0.3cm}} = R(q)$$

$$\boxed{\hspace{0.3cm}} = PS(q)$$

$$PS(q) = R(q) - V(q)$$

$$R(q) = pq$$

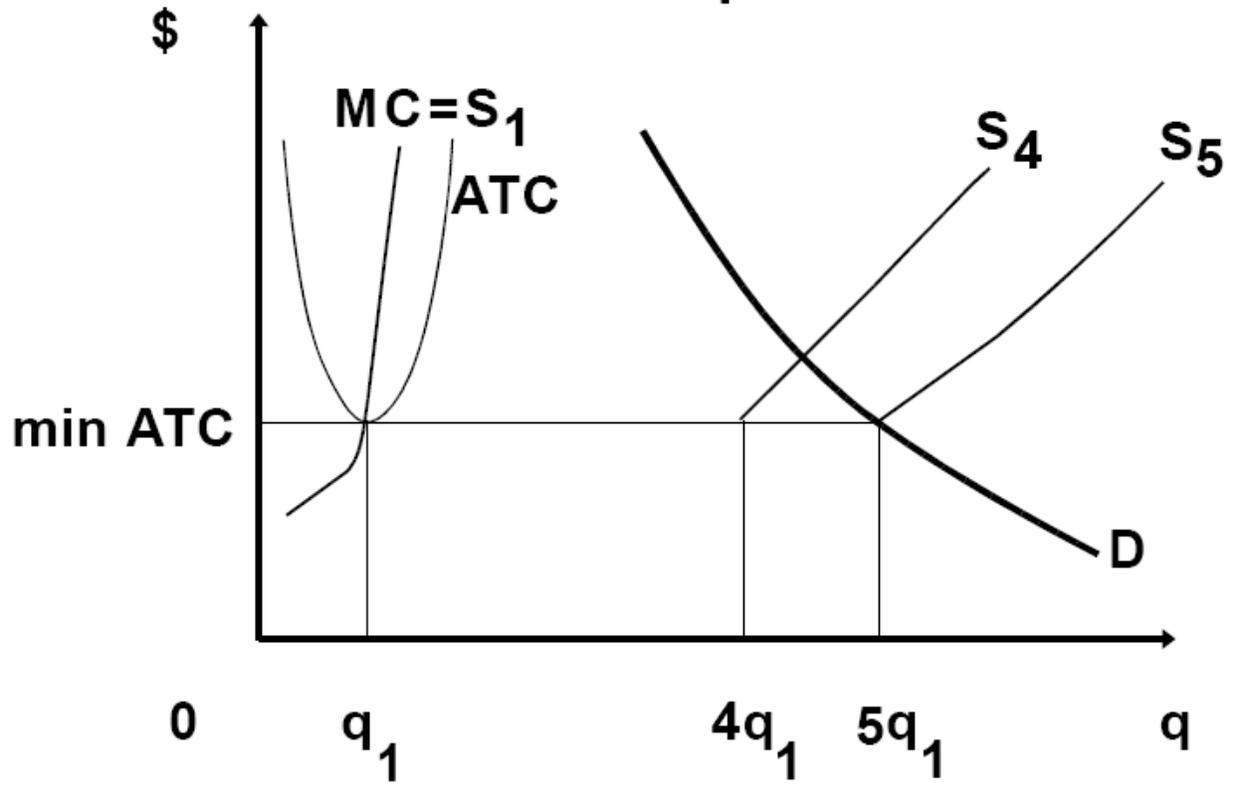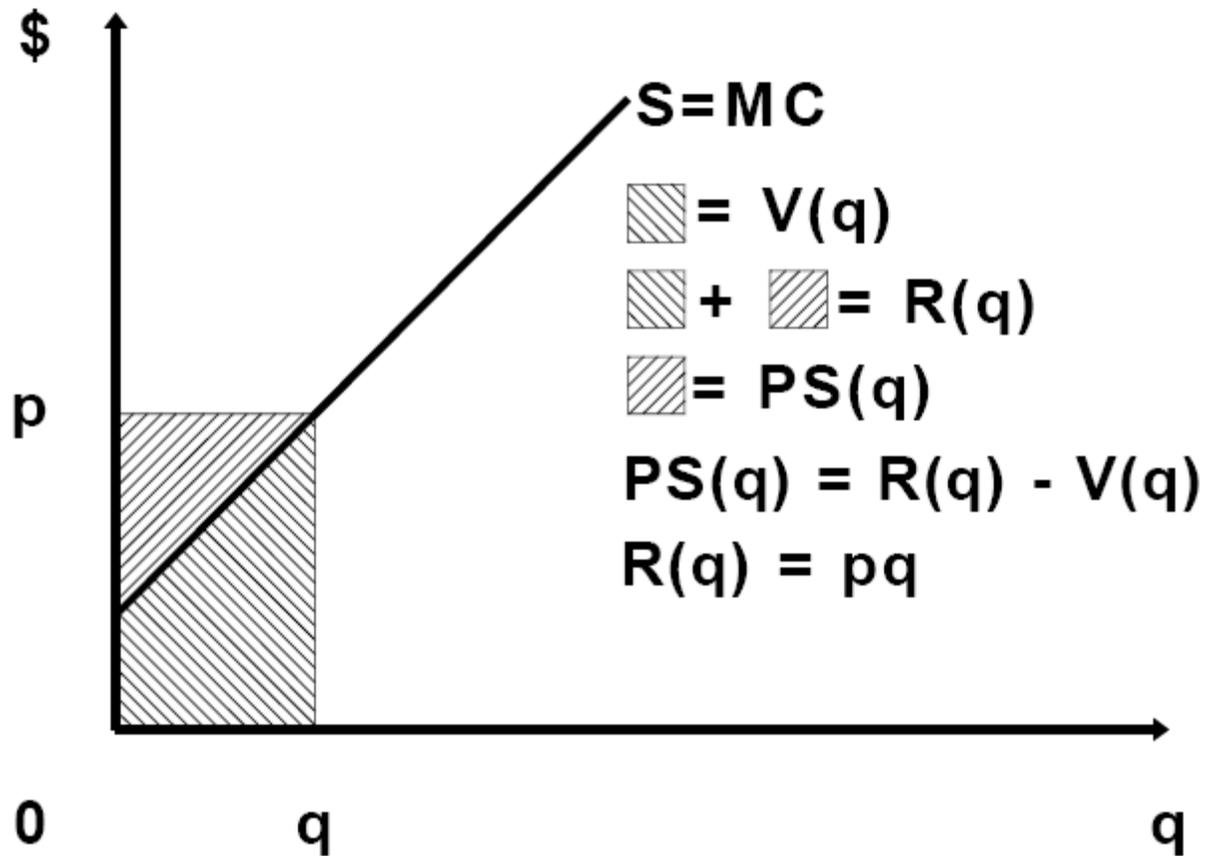**Figure 26**

39. **Producers surplus** is the difference between the revenue of the firm and its variable costs,

$$PS(q) = R(q) - V(q).$$

Variable costs, $V(q)$ can be represented by the area under the marginal cost curve (Figure 26). $V(q)$ is the minimum revenue a firm is willing to accept in

the short run to produce q.

Total surplus *(Figure 27) is the sum of consumers and producers surplus,*

$$TS(q) = CS(q) + PS(q).$$

Since consumers surplus is the area under the demand minus revenue,

$$CS(q) = A(q) - R(q),$$

total surplus is

$$TS(q) = A(q) - R(q) + R(q) - V(q) = A(q) - V(q).$$

It is maximized at the quantity q_c where the marginal cost curve intersects the demand curve.

Total surplus
(= consumers' + producers' surplus)

$\square = V(q_1)$

$\square + \square = A(q_1)$

$\square = TS(q_1)$

$TS(q) = CS(q) + PS(q)$

$CS(q) = A(q) - R(q)$

$PS(q) = R(q) - V(q)$

$TS(q) = A(q) - V(q)$

$\boxtimes$ = Dead Weight Loss

Figure 27
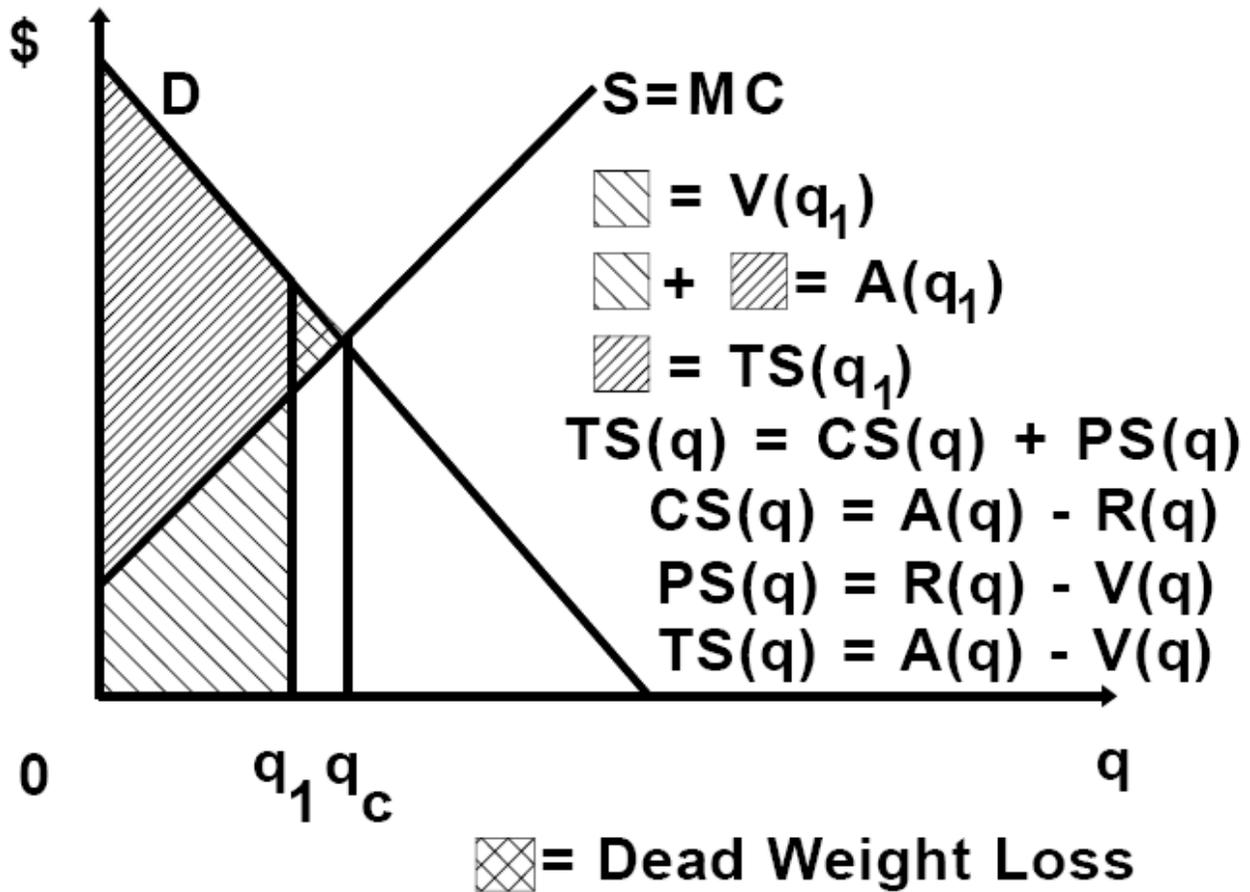
Specific Taxation and Surplus Loss
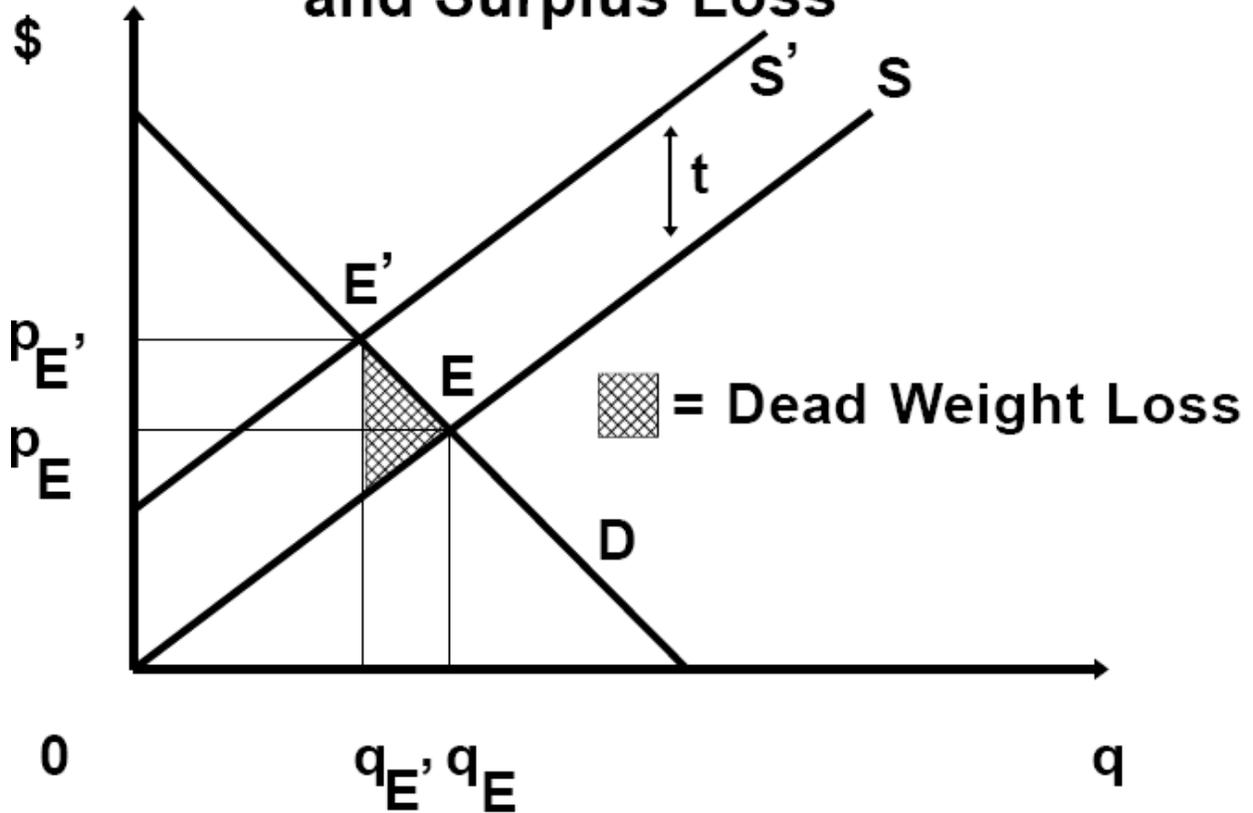
Figure 28

40. If the market is not perfectly competitive, each firm faces a downward sloping demand. Since revenue is

$$R(q) = qp(q),$$

marginal revenue is

$$MR(q) = p + q(dp/dq) = p[1 + (q/p)(dp/dq)] = p(1 + 1/\varepsilon) = p(1 - 1/|\varepsilon|),$$

where $|\varepsilon|$ is the elasticity of the demand *faced by the firm*. That is, marginal

revenue is intimately related to elasticity. Note that when the demand is

horizontal, or *perfectly elastic*, $|\varepsilon| = \infty$, the formula for marginal revenue

gives $MR = p(1 - 0) = p$. For a downward sloping demand, see Figure 30, and

the table below.

Demand, revenue, marginal revenue, and elasticity

Figure 29

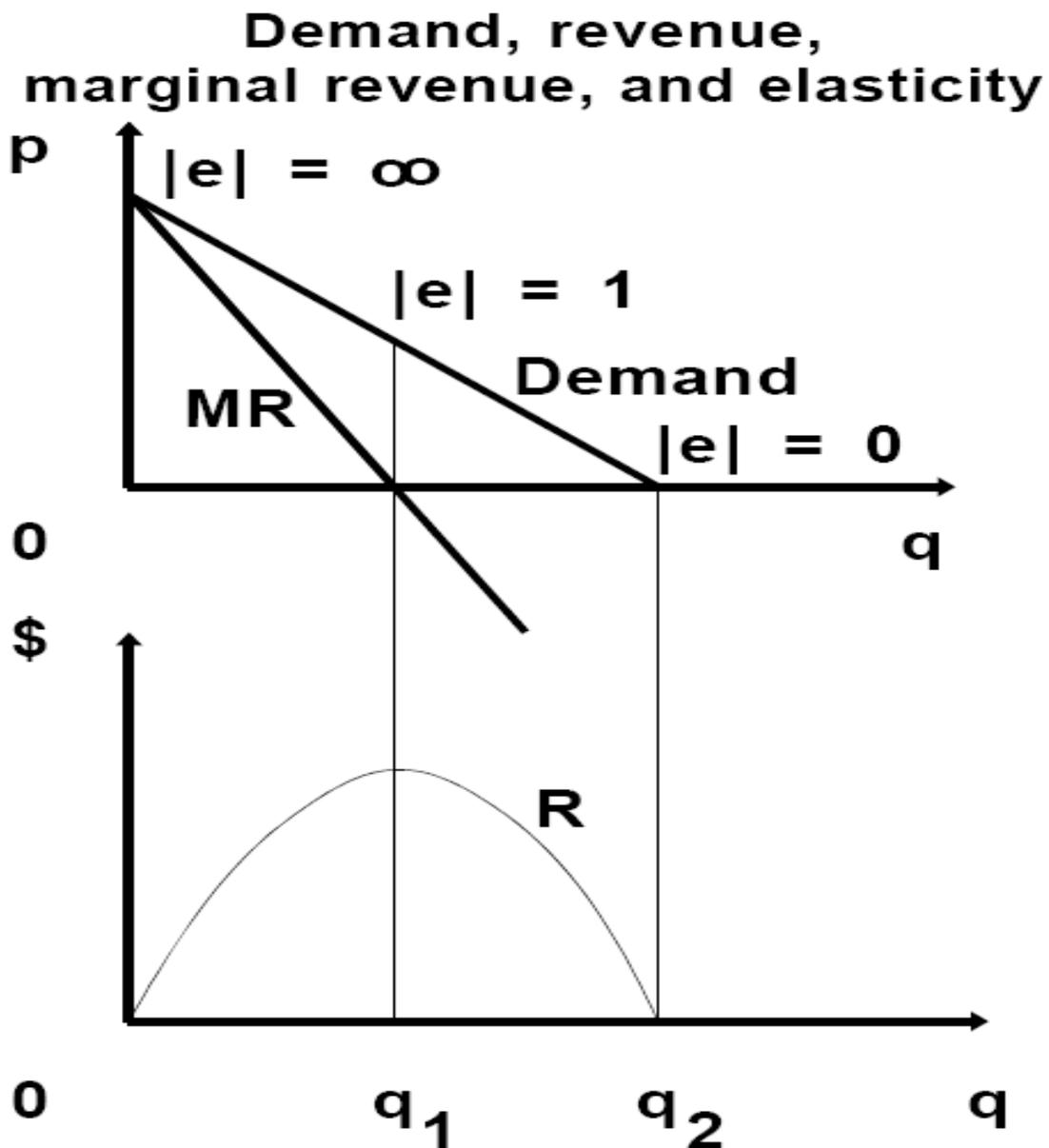| **Demand curve**	**Marginal revenue**
Elastic, $\lvert \varepsilon \rvert > 1$	$MR > 0$
Unit-elastic, $\lvert \varepsilon \rvert = 1$	$MR = 0$
Inelastic, $\lvert \varepsilon \rvert < 1$	$MR < 0$

41. **Monopoly**. A monopolist produces q_m that solves $MR(q_m) = MC(q_m)$.

Because $MR(q) = p(1 - 1/\lvert \varepsilon \rvert) < p$, for the same level of quantity, the marginal

revenue to a monopolist is lower than to a competitive firm. As a result, *a*

monopolist produces a lower level of output than a competitive firm,

$$q_m < q_c,$$

where q_c is at the intersection of marginal revenue and marginal cost. The

lower quantity produced by the monopolist corresponds to a higher price,

$$p_m > p_c.$$

Monopoly is inefficient and it creates a surplus loss (Figure 32). This is

because monopoly production is lower than the surplus-maximizing production,

$q_m < q_c$. Each unit q between q_m and q_c costs less to produce that the price that

someone is willing to pay for it as seen from the demand curve, *i.e.*, $MC(q) <$

$p(q)$. Therefore, *it is socially beneficial for these units to be produced*, but the

monopolist does not produce them since this is not in his *private* interest. The

inefficiency due to monopoly is called **dead weight loss** and is measured in $

terms by the triangle enclosed by the demand curve, the MC curve and line q_m.
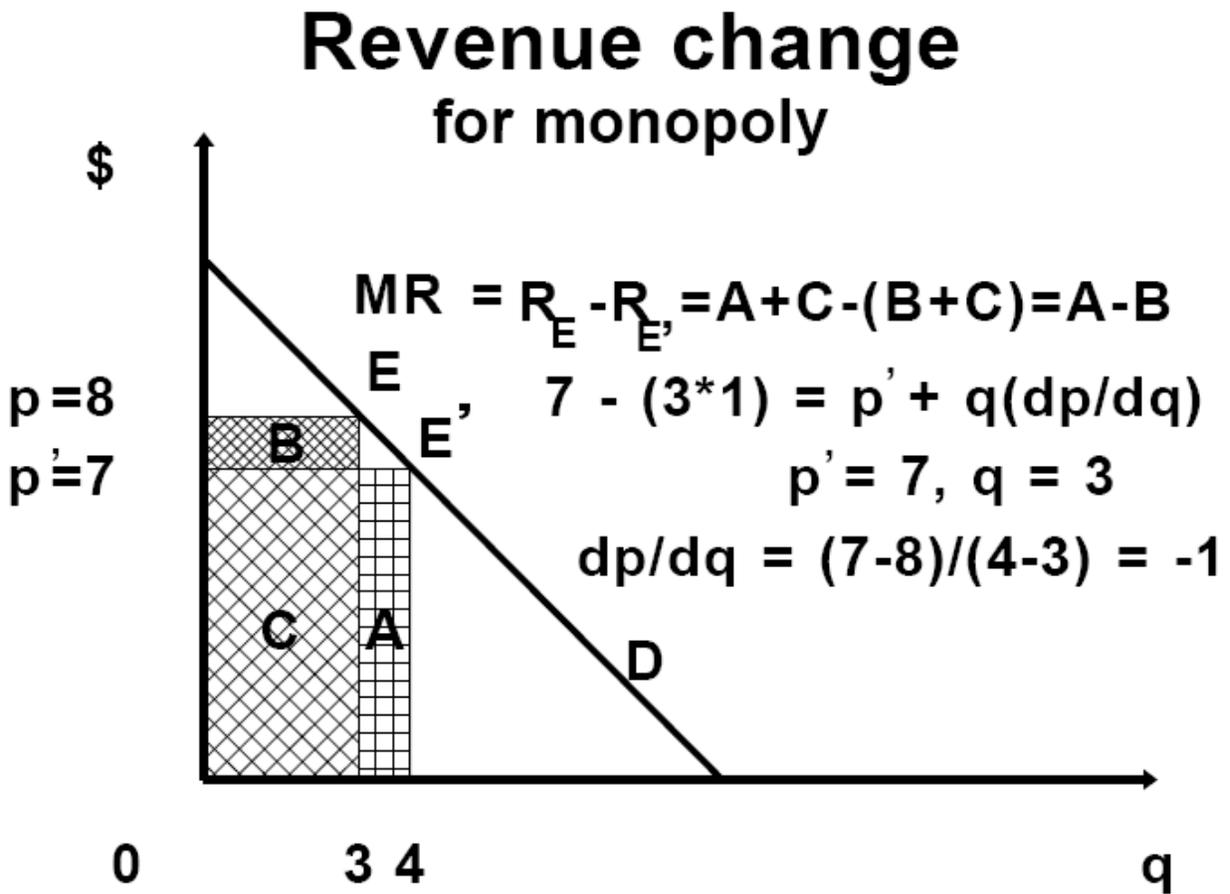
Revenue change
for monopoly

$$MR = R_E - R_{E'} = A + C - (B+C) = A - B$$

$$7 - (3*1) = p' + q(dp/dq)$$

$$p' = 7, q = 3$$

$$dp/dq = (7-8)/(4-3) = -1$$

p=8

p'=7

$

E

E'

B

E'

C

A

D

0   3 4   q

Figure 30

Monopoly and profits
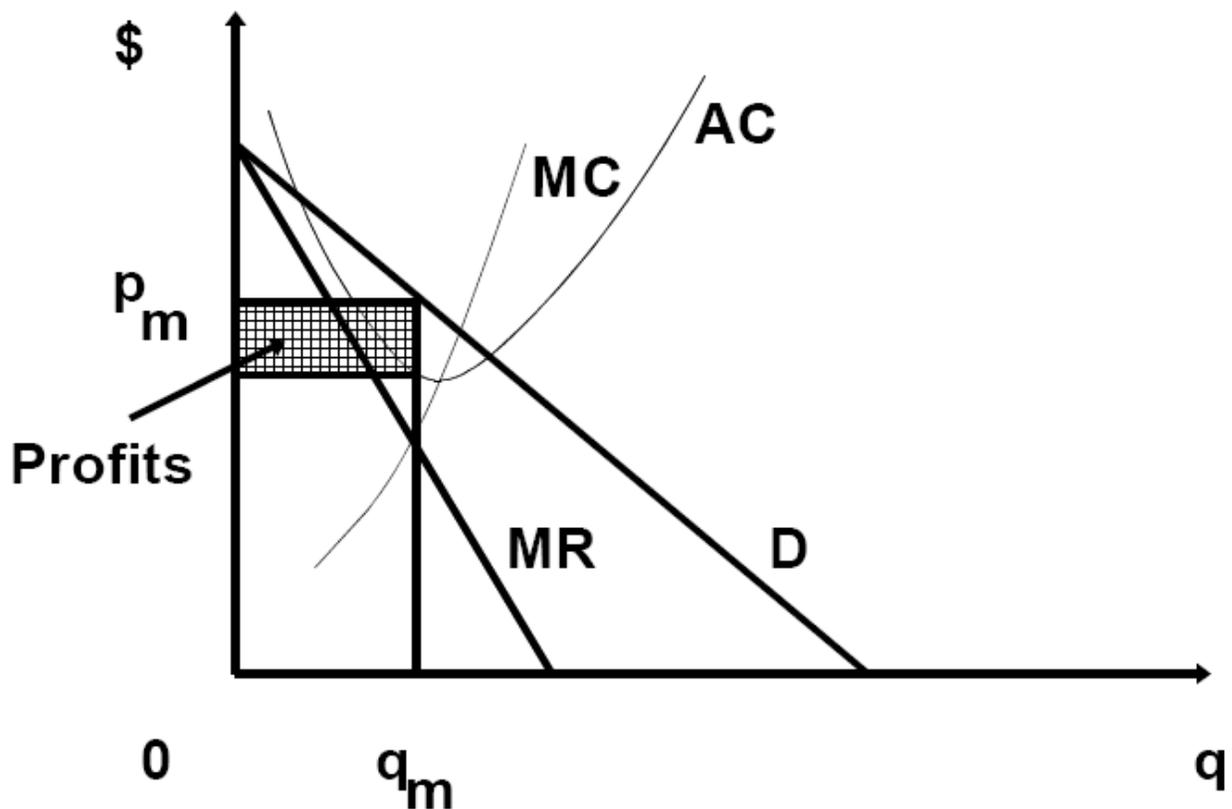
Figure 31

42. **Price discrimination** occurs when different units of the same product are sold at different prices to different consumers, or even to the same consumer.

Examples: 1. A unit of electricity (a KiloWattHour, KWH) is sold at a lower price to a manufacturer than to residential customer. 2. Your Con Edison Bill has a lower price for each of the first 240 KWH than for units above 240. 3. Japanese car manufacturers have been accused of selling in the US market at a lower price than in their home market. 4. Residential local telephone service

Monopoly and surplus loss
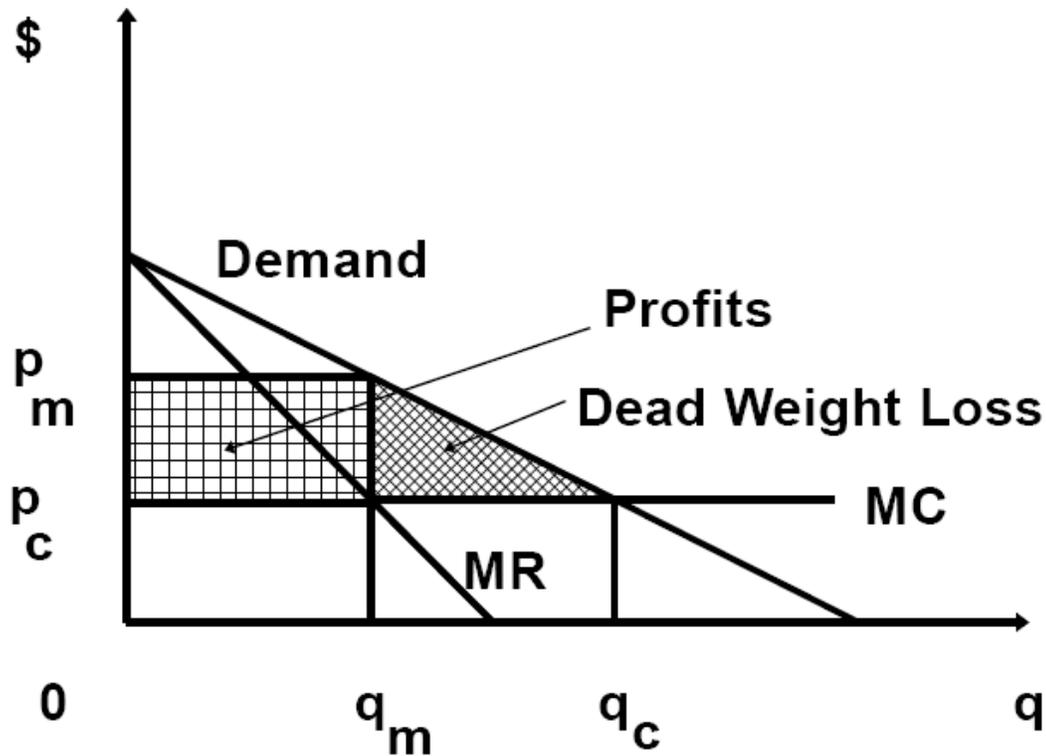
Figure 32

costs less than business local telephone service. 5. Doctors sometimes charge

lower prices to poorer patients. 6. Single product loyalty discounts for high

levels of consumption, as in the airlines frequent flyer programs.

There are two essential requirements for price discrimination to work

effectively. They are *sorting* and *no arbitrage*. The seller must be able to sort

the buyers in groups according to their willingness to pay for the good.

Otherwise, he does not know to whom to offer the high or low price. The seller

must also make sure that the buyers who were able to buy at a low price do not go around and sell what they bought to other buyers who are only offered a high price by the seller. Such activity is called *arbitrage* and would result in a single price in the market. For example, price discrimination can work well in the market for medical services, since it is impossible to transfer the service from one person to the other. However, price discrimination is very hard to work in the stock market. The low price recipients would sell the stocks to the others, and arbitrage would equalize the price.

43. In *perfect price discrimination* each consumer is offered a price equal to his willingness to pay for the good. Each buyer's consumer surplus is zero. Producers appropriate the whole surplus. In *imperfect price discrimination* consumers are split in groups according to willingness to pay, but the sorting is not perfect. For example, the local TV cable monopoly can charge a different price to "uptown" customers than to downtown customers.

Another example of price discrimination is a *two-part tariff*. For example, your Verizon bill includes a fixed fee which does not depend on the number of calls, on top of the price you pay for each phone call. Disneyland used to charge a fixed fee at the entrance plus a fee per ride.

Bundling is a special case of price discrimination, in which two goods are sold together (as a bundle) at a lower price than separately. For example, a computer system (composed of a CPU, a video monitor, a hard drive, a keyboard, and Windows) is sold at a lower price than the sum of the *a la carte* prices of the individual components. Bundled components do not have to be related (e.g., Polaroid-TWA). Additionally, in a more complex bundling contract, a company may offer a loyalty discount on a bundle of products attempting to leverage its power in one market to other markets. For example: a contract might say: if you buy 90% of all your needs from company A, you get a discount.

44. **Perfect Price Discrimination**. Under perfect price discrimination, each unit is sold at the willingness to pay for that unit of the person who is buying it. Therefore the revenue of the perfectly price discriminating monopolist is the whole area under the demand up to his level of production q,

$$R(q) = A(q).$$

Perfect Price Discrimination
by a Monopolist

Figure 34
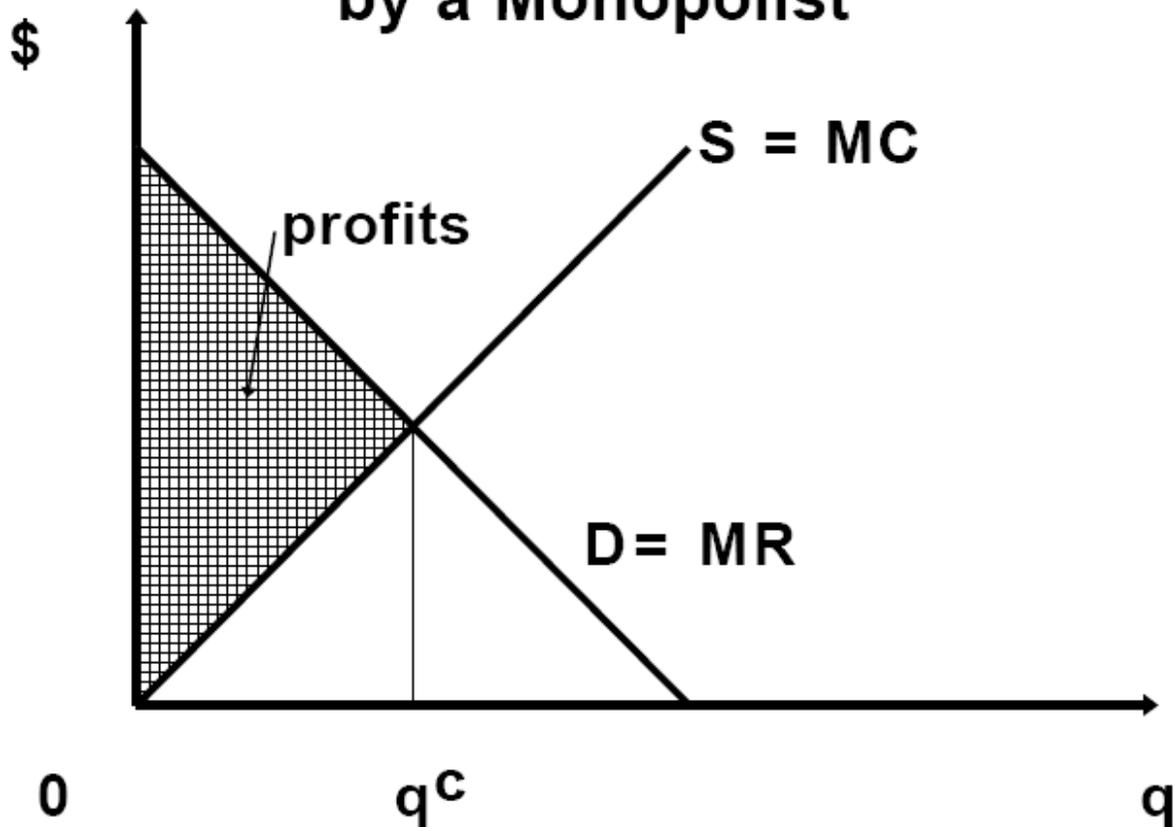
His marginal revenue is the revenue generated by the sale of the last unit, *i.e.*,

the price of this last unit q on the demand curve,

$$MR(q) = p(q).$$

In maximizing profits, the perfectly discriminating monopolist is setting MC(q)

= MR(q). But since MR(q) = p(q), he sets MC(q) = p(q). Therefore *the*

perfectly price discriminating monopolist produces the competitive output level

57

q_c. Remember, in this case consumers surplus is zero, and profits are equal to total surplus. Despite this distributional inequality, perfect price discrimination is *efficient* because it maximizes total surplus.

45. **Game theory**. *Games* describe situations where there is potential for conflict and for cooperation. Many business situations, as well as many other social interactions have both of these such features.

Example 1: Company X would like to be the only seller of a product (a monopolist). The existence of a competing firm Y hurts the profits of firm X. Firms X and Y could cooperate, reduce total production, and increase profits. Or they could compete, produce a high quantity and realize small profits. What will they do?

Example 2: Bank 1 competes with bank 2 for customers. Many of their customers use Automated Teller Machines (ATMs). Suppose that each bank has a network of its own ATM machines which are currently available only to its customers. Should bank 1 allow the customers of the other bank to use its ATMs? Should bank 1 ask for reciprocity?

Example 3: Computer manufacturer 1 has a cost advantage in the production of network "cards" (interfaces) of type 1. Similarly manufacturer 2 has an

advantage in network "cards" of type 2. If they end up producing cards of different types, their profits will be low. However, each firm makes higher profits when it produces the "card" on which it has a cost advantage. Will they produce "cards" of different types? Of the same type? Which type?

A **game in extensive form** is defined by a set of players, i = 1, ..., n, a **game tree**, **information sets**, **outcomes**, and **payoffs**. The game tree defines the sequence and availability of **moves** in every **decision node**. Each **decision node** is identified with the player that decides at that point. We assume there is only a finite number of possible moves at every node. Each branch of the tree ends at an event that we call an **outcome**. The utility associated with the outcome for every player we call his **payoff**. **Information sets** contain one or more nodes. They show the extent of knowledge of a player about his position in the tree. A player only knows that he is in an information set, which may contain more than one nodes. Information sets allow a game of simultaneous moves to be described by a game tree, despite the sequential nature of game trees. A game where each information set contains only one point is called a game of **perfect information**. (Otherwise it is of **imperfect information**.) For example, in the "Incumbent-Entrant" game (Figure 35), at every point, each

player knows all the moves that have happened up to that point. All the information sets contain only a single decision node, and the game is of perfect information. In the "Simultaneous Incumbent-Entrant" game (Figure 36), player I is not sure of player E's decision. He only knows that he is at one of the two positions included in his information set. It is as if players I and E move simultaneously. This is a game of imperfect information. Note that this small change in the information sets of player I makes a huge difference in what the game represents -- a simultaneous or a sequential decision process.

When the utility functions associated with the outcomes are known to both players, the game is of **complete information**. Otherwise it is of **incomplete information**. You may not know the opponents' utility function. For example, in a price war game, you may not know the value to the opponent firm (or the opponent manager) of a certain loss that you can inflict on them.

46. A **game in normal form** is a summary of the game in extensive form. This is facilitated by the use of **strategies**. *A strategy for player i defines a move for this player for every situation where player i might have to make a*
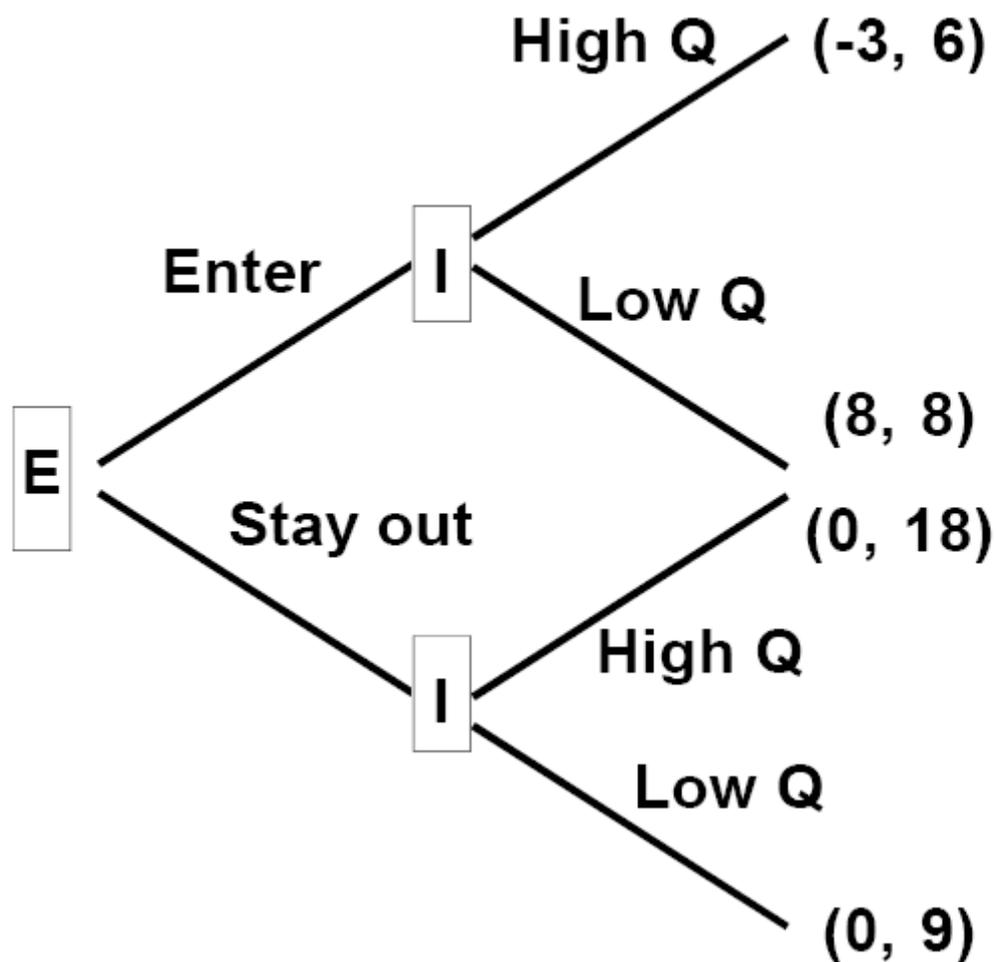
Incumbent-Entrant game

Figure 35

move in the game. A strategy of player i is denoted by s_i, and it belongs in the set of available strategies of player i, S_i. By its nature, a strategy can be very complicated and long. For example, a strategy for white in chess would

Simultaneous Incumbent-Entrant game

Figure 36
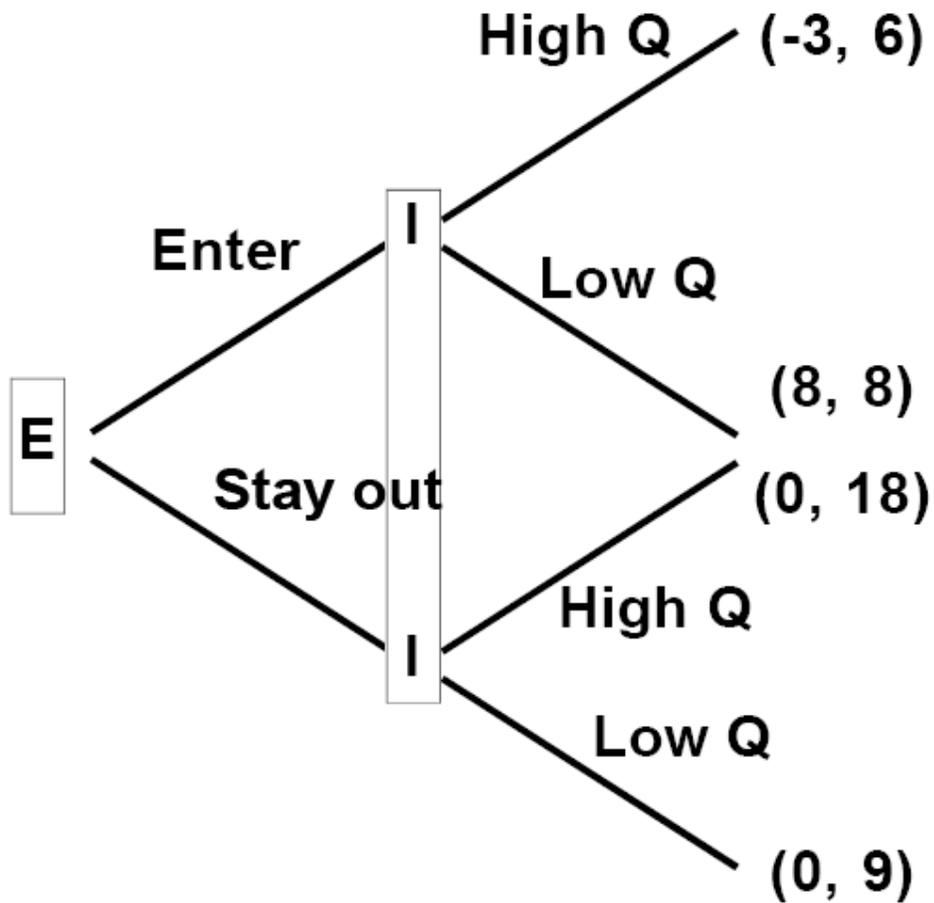
have to specify the opening move, the second move conditional on the 20

alternative first moves of the black, the third move conditional on the many (at

least 20) alternative second moves of the black, and so on. The advantage of

using strategies is that, once each player has chosen a strategy, the outcome

(and the corresponding payoffs) are immediately specified. Thus, the analysis

of the game becomes quicker.

Example 1:   **"Simultaneous Incumbent-Entrant Game"**

		Player 2 (Incumbent)	
	Strategies	High Q	Low Q
Player 1 (Entrant)	Enter	(-3,6)	(8, 8)
	Stay out	(0, 18)	(0, 9)

Strategies for Player 1: Enter, Stay out
Strategies for Player 2: High Q, Low Q

Example 2:   **"Prisoners' Dilemma Game"**

		Player 2	
	Strategies	silence	Talk
Player 1	Silence	(5, 5)	(0, 6)
	Talk	(6, 0)	(2, 2)

Strategies for player 1: Silence, Talk.
Strategies for player 2: silence, talk.

Prisoners' Dilemma

Figure 37

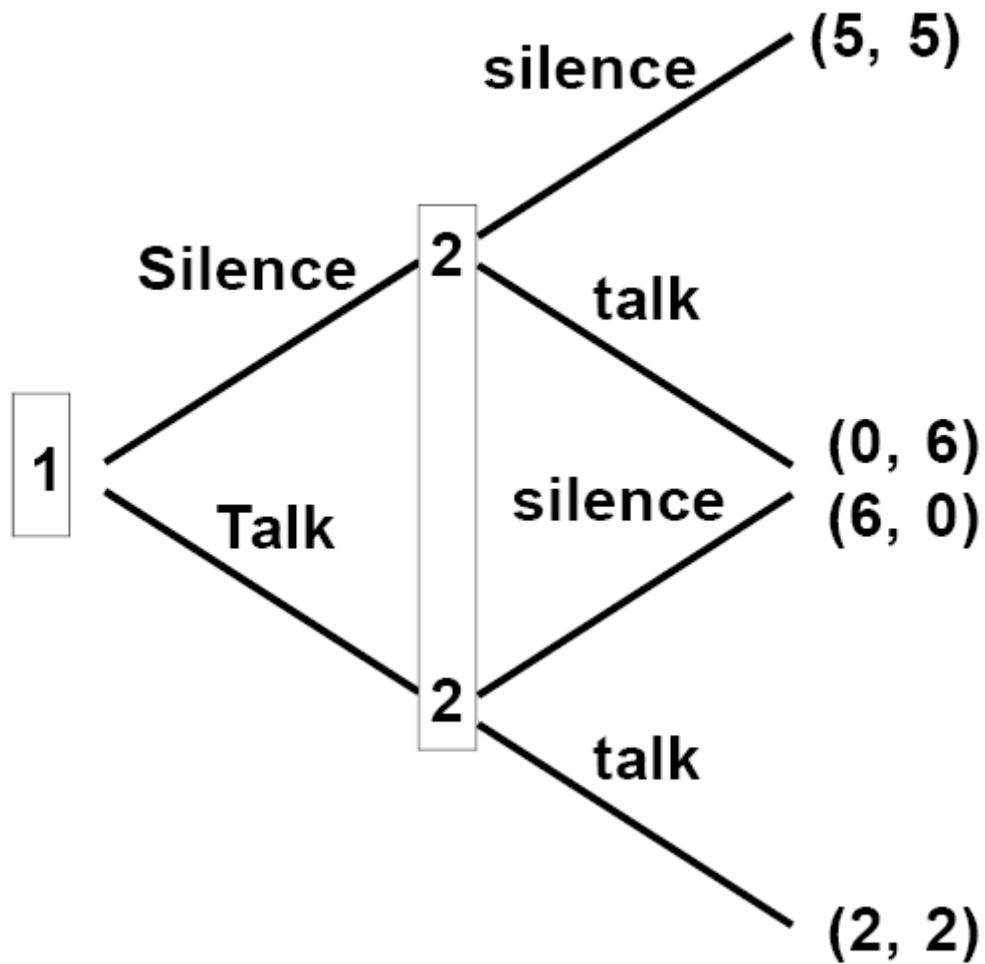47. **Non-cooperative equilibrium**. A pair of strategies (s_1, s_2) is a **non-cooperative equilibrium** if and only if each player has no incentive to change his strategy provided that the opponent does not change his strategy. No player has an incentive to unilaterally deviate from an equilibrium position.

Looking at the Prisoners' Dilemma, we see that if player 2 plays "silence" and is

expected to continue playing this strategy, player 1 prefers to play "Talk", since he makes 6 instead of 5 in the payoff. Since player 1 wants to deviate from it, (Silence, silence) is **not** a non-cooperative equilibrium. If player 2 plays "talk" and is expected to continue playing it, player 1 prefers to play "Talk", since he makes 2 instead of 0 in the payoff. Therefore (Talk, silence) is **not** a non-cooperative equilibrium. Finally, given that player 1 plays "Talk", player 2 prefers to play "talk" since he gets 2 instead of 0. Since both players prefer not to deviate from the strategy they play in (Talk, talk) if the opponent does not deviate from his strategy in (Talk, talk), this **is** a non-cooperative equilibrium. In the "simultaneous Incumbent-Entrant" game, if E enters, the incumbent prefers to play L because his payoff is 8 rather than 6. If the incumbent plays L, the entrant chooses to play Enter, because he prefers 8 to 0. Therefore **no player has an incentive to deviate from (Enter, Low Q), and it is a non-cooperative equilibrium**. In the same game, if the entrant chooses to stay out, the incumbent replies by producing a high quantity (H) because 18 is better than 9. And, if the incumbent produces a high Q, the entrant prefers to stay out because 0 is better than -3. Therefore **no player has an incentive to deviate from (Stay out, High Q), and it is also a non-cooperative equilibrium**. Note

that there are two equilibria in this game.

To find the equilibrium in the original sequential "Incumbent-Entrant" game of Figure 35, note that if player E enters, player I prefers Low Q, and ends up at (8, 8). If E stays out, I prefers High Q, and ends up at (0, 18). Seeing this, E chooses to enter because in this way he realizes a profit of 8 rather than of 0. **Therefore the non-cooperative equilibrium is at (Enter, Low Q)**, and both firms realize a profit of 8. Note that *the other equilibrium of the simultaneous game was eliminated*.

48.   **Dominant strategies**. In some games, no matter what strategy player 1 plays, there is a single strategy that maximizes the payoff of player 2. For example, in the Prisoners' Dilemma if player 1 plays "Silence", it is better for player 2 to play "talk"; and, if player 1 plays "Talk", it is better for player 2 to play "talk" again. Then "talk" is a **dominant strategy** for player 2. In the same game, note that "Talk" is a dominant strategy for player 1, because he prefers it no matter what player 2 plays. In a game such as this, where both players have a dominant strategy, there is an **equilibrium in dominant strategies**, where each player plays his dominant strategy. An equilibrium in dominant strategies is necessarily a non-cooperative equilibrium. (Why? Make sure you

understand that no player wants to unilaterally deviate from a dominant strategy equilibrium.)

49.   There are games with no equilibrium in dominant strategies. For example, in the simultaneous incumbent-entrant game, the entrant prefers to stay out if the incumbent plays "H". However, the entrant prefers to enter if the incumbent plays "L". Since the entrant would choose a different strategy depending on what the incumbent does, the entrant does not have a dominant strategy. (Similarly, check that the incumbent does not have a dominant strategy.) Therefore in the simultaneous incumbent-entrant game there is no equilibrium in dominant strategies.

50.   **Strategic Trade Policy**. In many industries firms face U-shaped average cost functions with minimum efficient scale of significant size. Then the industrial policy or the trade policy of a country can have very significant impact on the entry of firms and the existence of an industry in a country. Consider an entry game between Boeing and Airbus with the following profit matrix.

Entry in Airplane Manufacturing Without Subsidies

		Player 2 (Airbus)

	Strategies	enter	don't enter
Player 1 (Boeing)	Enter	(-100, -100)	(500, 0)
	Don't Enter	(0, 500)	(0, 0)

This game has two non-cooperative equilibria. In each one of them, one firm enters and the other does not. However, if Airbus received a subsidy of 200 from France the profit matrix gets transformed into:

Entry in Airplane Manufacturing with Airbus Subsidy Only

		Player 2 (Airbus)	
	Strategies	enter	don't enter
Player 1 (Boeing)	Enter	(-100, 100)	(500, 0)
	Don't Enter	(0, 700)	(0, 0)

Now the "enter" strategy becomes dominant for Airbus, and the only non-cooperative equilibrium is when Airbus enters and Boeing stays out. If the US were to subsidize Boeing by 200 in the absence of an Airbus subsidy, in the new matrix the "enter" strategy would now be dominant for Boeing and the only non-cooperative equilibrium would have Boeing entering and Airbus

staying out. Of course, each country could each subsidize its company.

Entry in Airplane Manufacturing with Two Subsidies

		Player 2 (Airbus)	
	Strategies	enter	don't enter
Player 1 (Boeing)	Enter	(100, 100)	(700, 0)
	Don't Enter	(0, 700)	(0, 0)

Then both firms have a dominant strategy to enter. The public is the loser in this case, because the market is forced to have more firms than is efficient.

51. **Best replies**. Player 1's **best reply** to strategy s_2 of player 2 is defined as the best strategy that player 1 can play against strategy s_2 of player 2. For example, in the simultaneous incumbent-entrant game, the best reply of the entrant to the incumbent playing "High Q" is "Stay out". Similarly, the best reply of the entrant to the incumbent playing "Low Q" is "Enter". From the point of view of the incumbent, his best reply to the entrant's choice of "Enter" is "Low Q", and his best reply to the entrant's choice of "Stay out" is "High Q". Notice that at a non-cooperative equilibrium both players play their best replies to the strategy of the opponent. For example, at (Stay out, High Q), as we saw,

"Stay out" is a best reply to "High Q", and "High Q" is a best reply to "Stay out". This is no coincidence. At the non-cooperative equilibrium no player has an incentive to deviate from the strategy he plays. This means that his strategy is the best among all the available ones, as a reply to the choice of the opponent. This is just another way of saying that the player chooses the best reply strategy. Therefore **at equilibrium each player plays a best reply strategy**. This suggests that we can find equilibria by finding first the first reply strategies. We do this in the oligopoly games that we discuss next.

52.   **Oligopoly**. When few firms are interacting in a market, they have to take into account the effects of each one's actions on the others. We use **game theory** to analyze these **strategic** situations.

The simplest oligopoly model is due to Augustin **Cournot** (1838). There is a single homogeneous good with demand function $p(Q)$. Two competing firms, i = 1, 2, produce q_i each. $Q = q_1 + q_2$. The profit function of firm 1 is

$$\Pi_1 = q_1 p(q_1 + q_2) - C_1(q_1).$$

In game theoretic terms, this is a game where each player uses the quantity he produces as his strategy. According to Cournot, each firm assumes that (at equilibrium) its actions have no influence on the actions of the opponent.

Therefore we seek the non-cooperative equilibrium of the game.

Residual demand
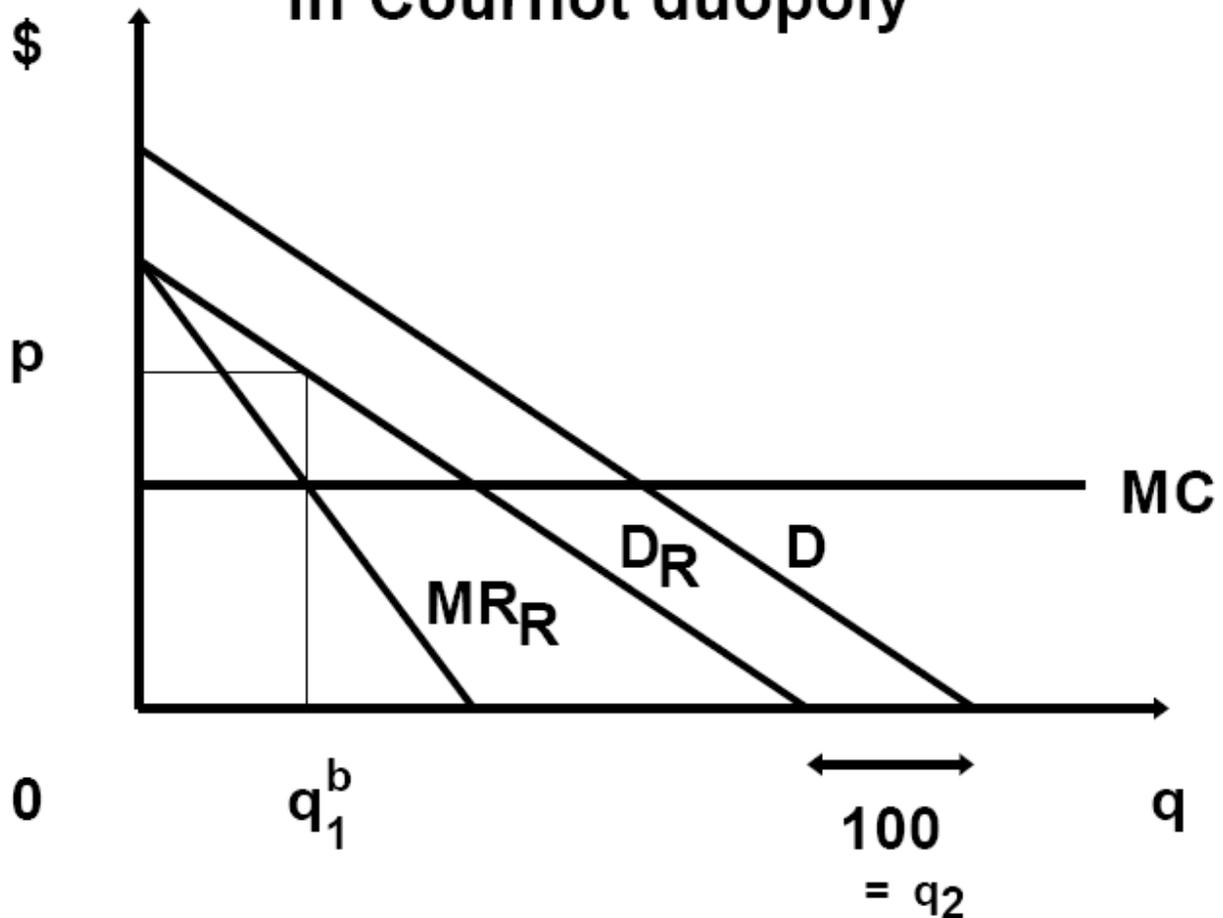## in Cournot duopoly

Figure 38

By assuming that firm 1 has no influence on the output of firm 2, Cournot

assumed that the **residual demand** facing firm 1 is just a leftward shift by q_2 of

the industry demand $p(Q)$ (Figure 38). Firm 1 is a monopolist on this residual

demand. For example, firm 2 produces $q_2 = 100$ units. The remaining demand

71

is drawn as D_R. Since firm 1 is a monopolist on D_R, we draw his marginal

revenue curve, MR_R, and find his optimal output q_1^b at the intersection of MC

and MR_R. This is the output that maximizes profits for firm

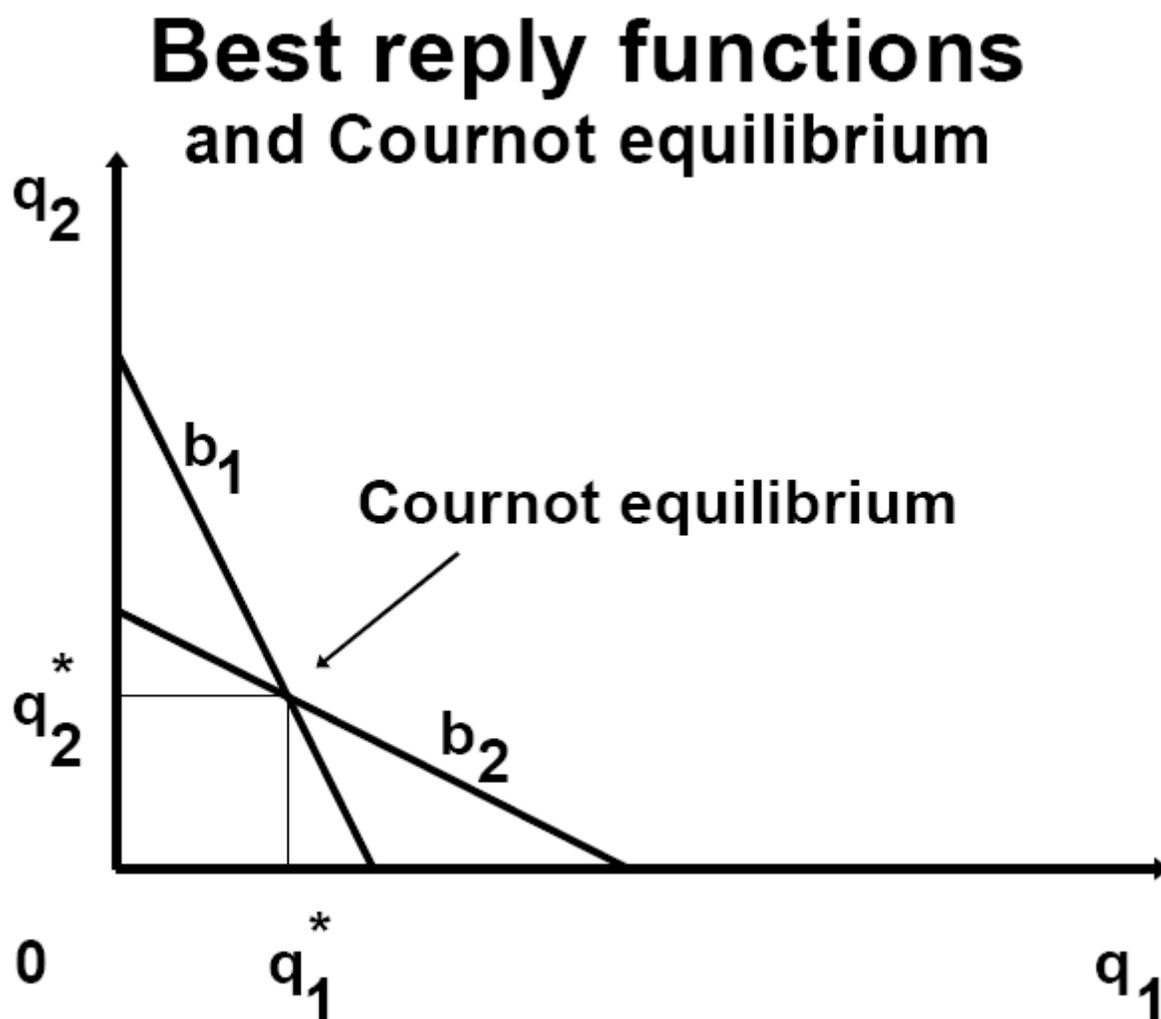## Best reply functions and Cournot equilibrium

Figure 39

1 provided that firm 2 produces $q_2 = 100$. This means that q_1^b is the best reply

of firm 1 to $q_2 = 100$. Similarly we can define the best reply of player 1 to

different levels of output q_2. We write this as

$$q_1 = b_1(q_2).$$

Similarly, solving the problem from the point of view of firm 2, we can derive the best reply of firm 2 to levels of output of firm 1,

$$q_2 = b_2(q_1).$$

The intersection of the two best reply functions defines the non-cooperative equilibrium, (q_1^*, q_2^*). See Figure 39.

53.   **Example of Cournot duopoly**.

$P = 120 - Q$. $Q = q_1 + q_2$. Zero costs. Profits of firms 1 and 2 are

$$\Pi_1 = q_1(120 - q_1 - q_2), \quad \Pi_2 = q_2(120 - q_1 - q_2).$$

To find the best reply of firm 1, we maximize Π_1 keeping q_2 constant:

$$d\Pi_1/dq_1 = 120 - q_1 - q_2 - q_1 = 0.$$

Solving for q_1 gives us the best reply for player 1:

$$q_1 = (120 - q_2)/2. \qquad\qquad (*)$$

Similarly, to find the best reply of firm 2, we maximize Π_2 keeping q_1 constant:

$$d\Pi_2/dq_2 = 120 - q_1 - q_2 - q_2 = 0.$$

Solving for q_2 gives us the best reply for player 2:

$$q_2 = (120 - q_1)/2. \qquad\qquad (**)$$

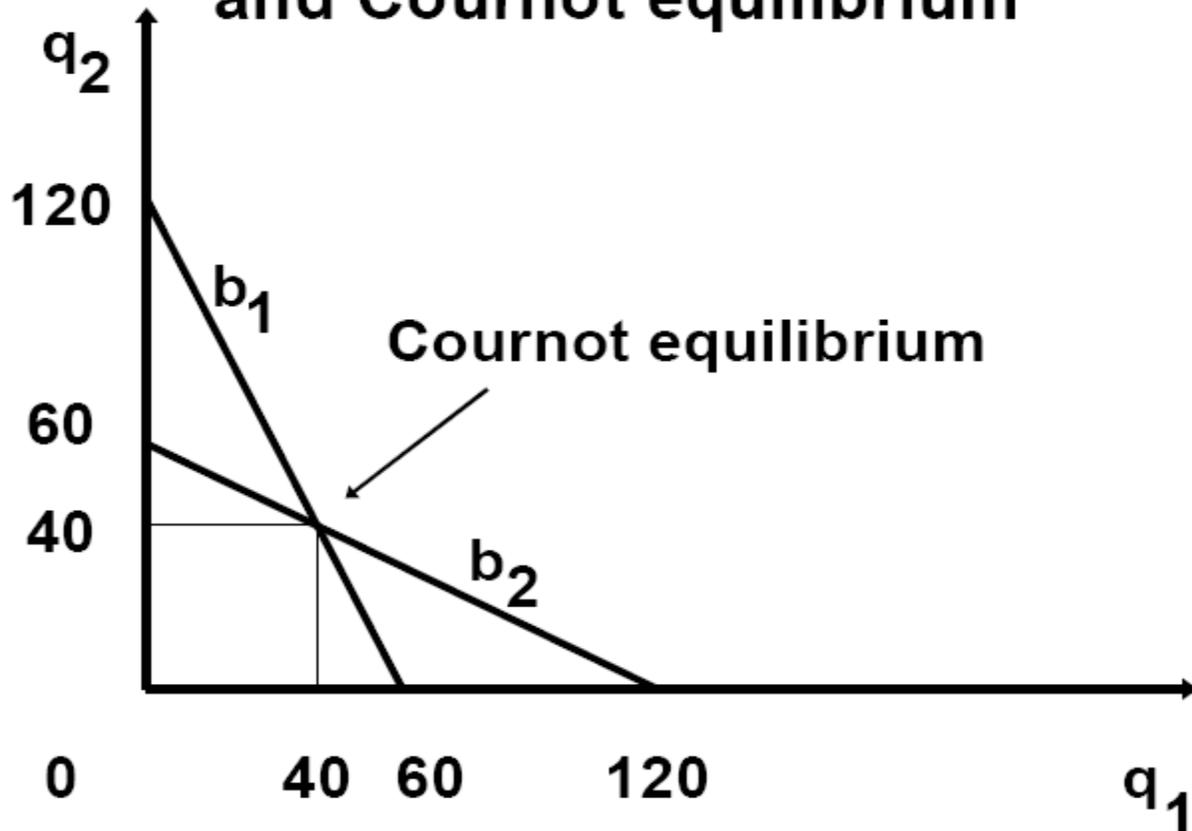**Best reply functions and Cournot equilibrium**

Figure 40

The best replies are pictured on Figure 40. Their intersection is the non-cooperative equilibrium. We find it by solving (*) and (**) together:

$$q_1 = (120 - q_2)/2 = [120 - (120 - q_1)/2]/2 = (60 + q_1/2)/2 = 30 + q_1/4,$$

$$\textit{i.e.,} \ 3q_1/4 = 30, \ \textit{i.e.,} \ q_1 = 40.$$

Substituting back at (**) we get $q_2 = 40$. Therefore the non-cooperative

equilibrium is $q_1^* = 40$, $q_2^* = 40$. Total production is $Q = 80$. Price is now $P =$

120 - 80 = 40. Profits are $\Pi_1 = \Pi_2 = 1600$. Total industry profits are 3200.

54. **Comparison of Monopoly and Duopoly**

Two colluding firms try to maximize joint (industry) profits. This is exactly what a monopolist would do. Joint profits are

$$\Pi(q_1, q_2) = \Pi_1 + \Pi_2 = q_1 P(q_1 + q_2) + q_2 P(q_1 + q_2) =$$

$$(q_1 + q_2)P(q_1 + q_2) = (q_1 + q_2)(120 - q_1 - q_2).$$

Maximizing with respect to q_1 and q_2 we have

$$0 = \Delta\Pi/\Delta q_1 = (120 - q_1 - q_2) - (q_1 + q_2) = 120 - 2(q_1 + q_2),$$

i.e., $q_1^M + q_2^M = 60$, which we know is the monopoly output $Q_M = 60$. Price is $P^M = 120 - 60 = 60$. Total equilibrium profits of collusion are

$$\Pi(q_1^M + q_2^M) = 60*60 = 3600.$$

Therefore, in comparison to the Cournot equilibrium, in monopoly price and profits are higher, while quantity is lower.

55. The following table summarizes the comparison between different industrial structures, also seen in Figure 41.

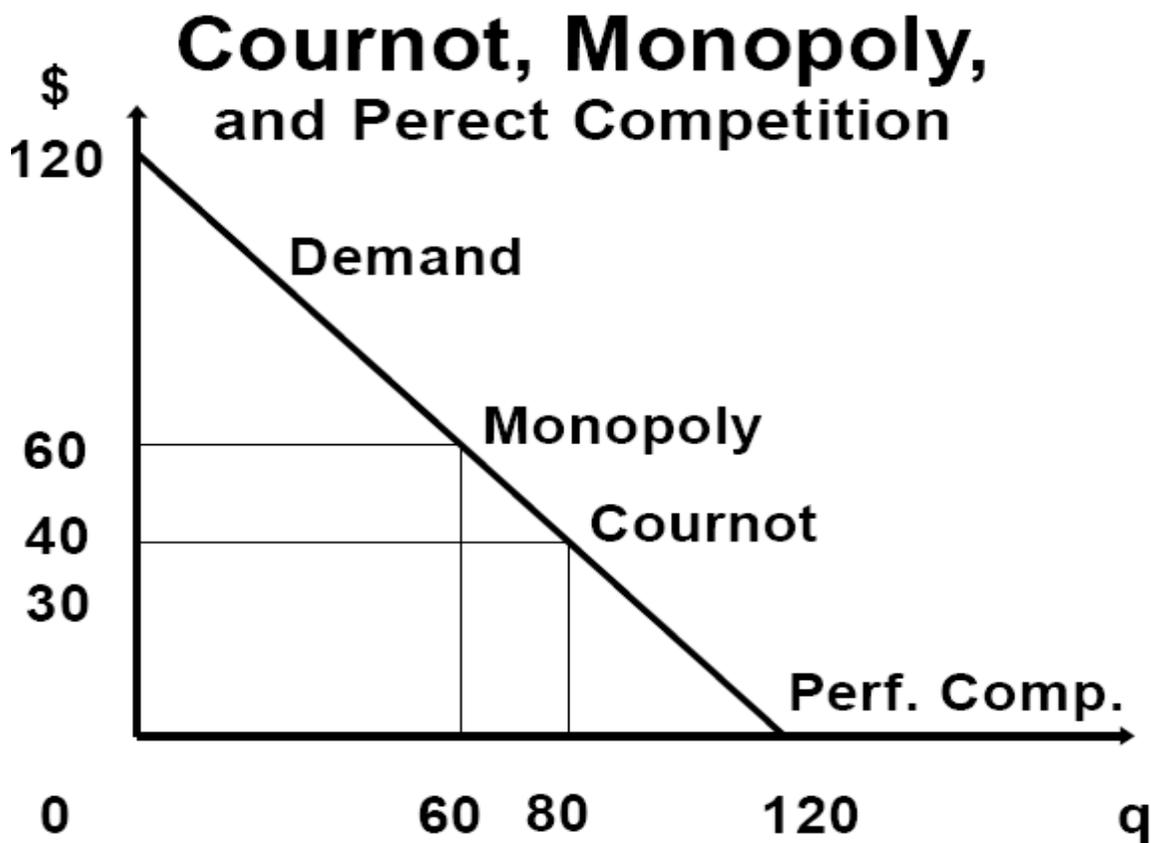| Perfect Competition	Cournot Duopoly	One-Price Monopoly	Perfect Price Discriminating Monopoly
P = 0	P = 40	P = 60	P varies
Q = 120	$q_1 = q_2 = 40$, Q = 80	Q = 60	Q = 120
$\Pi_i = 0$	$\Pi_1 = \Pi_2 = 1600$ $\Pi_1 + \Pi_2 = 3200$	$\Pi = 3600$	$\Pi = 7200$
CS = 7200	CS = 3200	CS = 1800	CS = 0
TS = 7200	TS = 6400	TS = 5400	TS = 7200
DWL = 0	DWL = 800	DWL=1800	DWL =0

Figure 41

56. Price Leadership, Cartels.

Here the leader sets the price p and the follower(s) accept the price and act as if they were in perfect competition. This is a common market behavior of cartels such as OPEC. They fix the price for the good and it is accepted by others in the industry outside the cartel.

Price leadership

In the figure the following labels appear:

$D(p)$

$S_f(p)$

$D_R(p) = D(p) - S_f(p)$

$q_t = q_f + q_L$
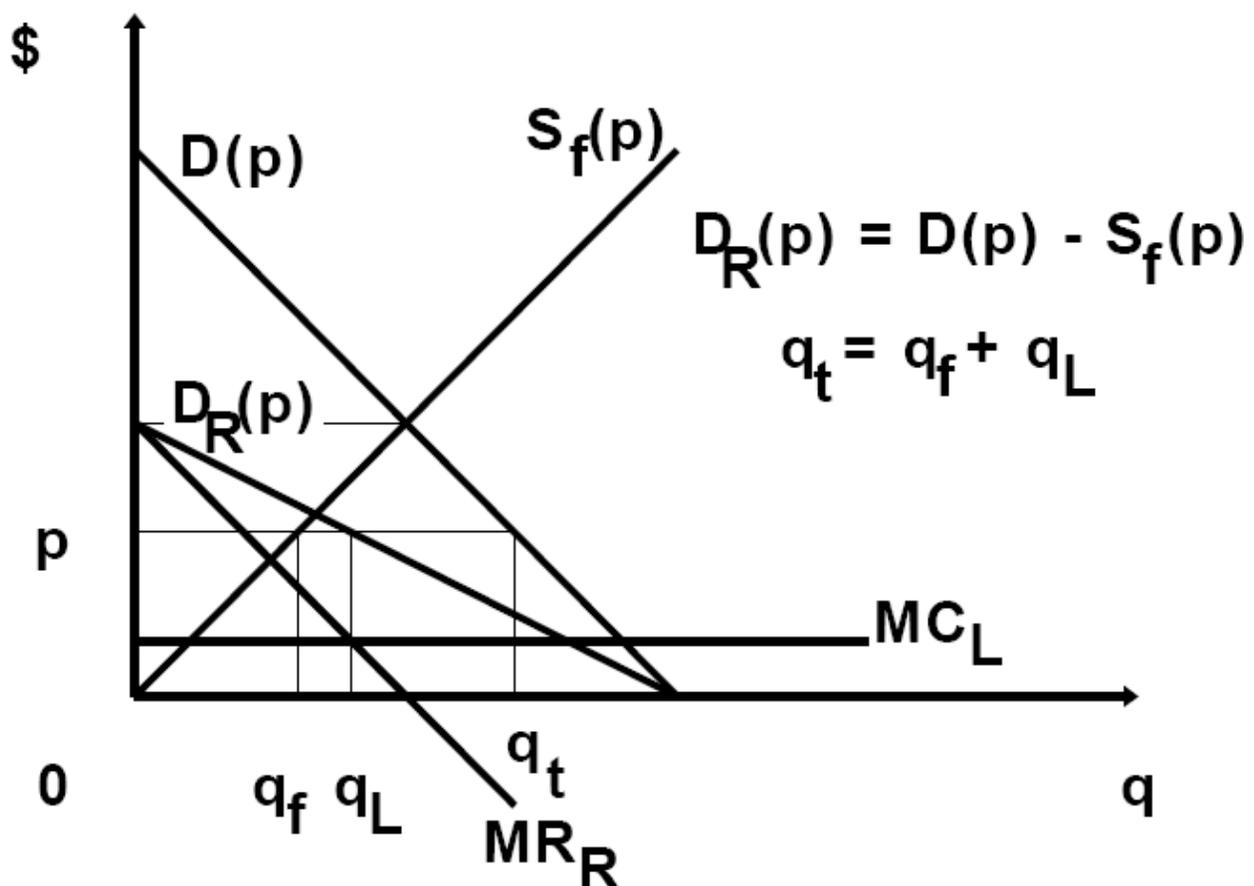
$D_R(p)$

MC_L

$q_f \quad q_L$

q_t

MR_R

Figure 42

The follower(s) are price-takers. Let their supply be $S_f(p)$. If the industry demand is $D(p)$, the leader faces the residual demand $D_R(p) = D(p) - S_f(p)$. The leader acts as a monopolist on the residual demand. Note that if the leader and the follower have the same marginal costs, since because the leader has to restrict his output (to achieve a high price) and the follower doesn't, the leader will have lower profits than the follower. This explains the incentive of a cartel member to leave the cartel. While inside the cartel, each member has to restrict production so that the cartel is able set a high price. If a firm moved out of the cartel, and the price remained the same, then the defector would produce more and make higher profits. The catch is that usually, when a firm leaves the cartel, the ability of the cartel to set the price is diminished. Therefore higher profits for the defector are not guaranteed, since he can sell more, but at a lower price. See Figure 42.

57.   **Monopolistic competition**. We started by studying **perfect competition** where no firm has any influence over the market price. We then studied its diametrically opposite market structure, *i.e.*, **monopoly**, where a single firm sets the market price. We proceeded to study **oligopoly** where few firms interact. Using **game theory** we analyzed the **strategic interactions**

among the few market participants. There is an extra category of market

structure that we haven't discussed yet. It is **monopolistic competition** and it

falls between oligopoly and perfect competition. There are a number of

competing firms, say above ten, but less than twenty five. In monopolistic

competition firms make zero profits. This distinguishes monopolistic

competition from oligopoly. At the same time, in monopolistic competition

Monopolistic Competition

$P_1 = AC(q_1)$
profits $= 0$

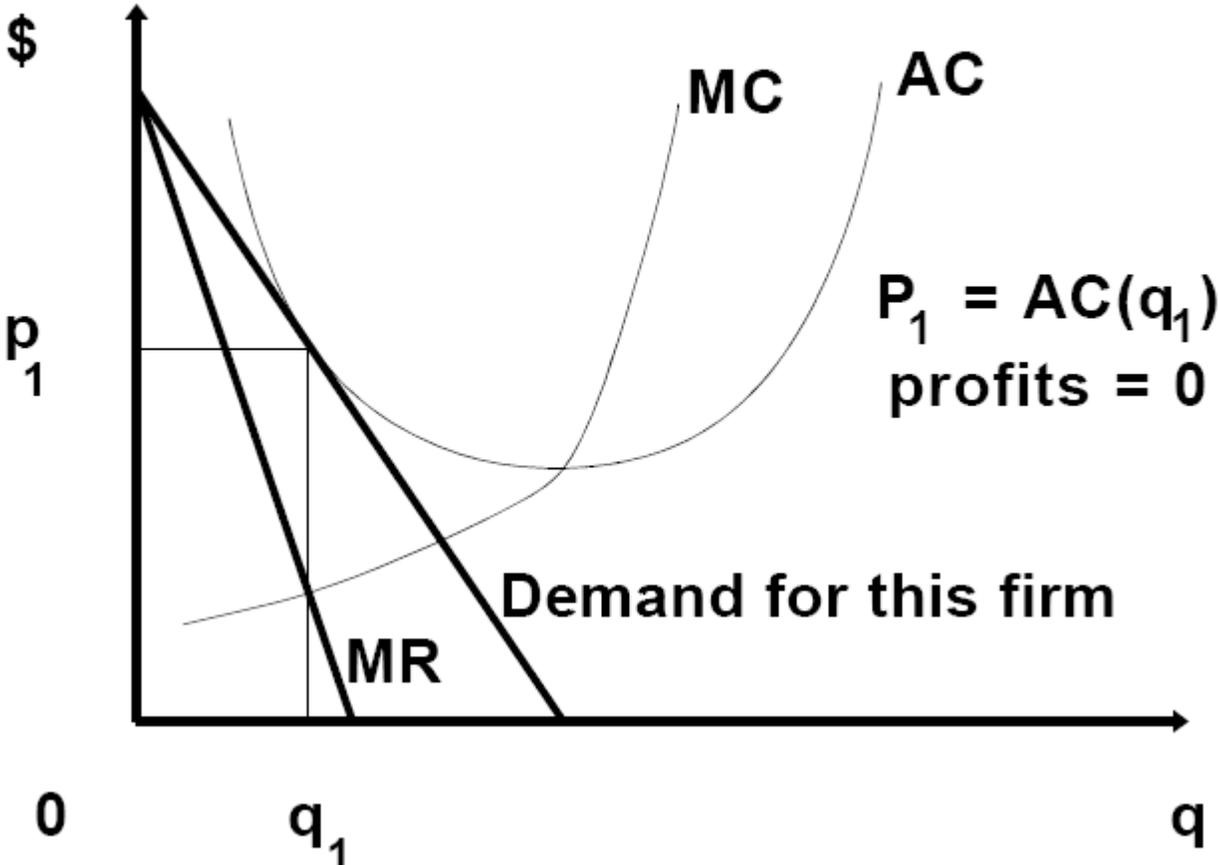Demand for this firm

MR

Figure 43

each firm faces a downward-sloping demand function. Therefore each firm has

some influence over price. This distinguishes monopolistic competition from

perfect competition. In monopolistic competition there are strategic interactions

among firms, but they are small. The following table summarizes the

similarities and differences of the various market structures.

Perfect Competition	Monopolistic Competition	Oligopoly	Monopoly
Zero profits		Positive Profits	
Non-strategic	Strategic but in limited scope	Strategic	Non-strategic
$n > 25$	$25 > n > 10$	$10 > n > 1$	$n = 1$

E.   <u>FACTOR MARKETS</u>

58.   **Equilibrium in factor markets**.

Every profit-maximizing firm uses a factor (say capital) until its **marginal**

expenditure in this factor, ME_K, is equal to the marginal revenue generated by

this factor, MRP_K, *i.e.*,

$$ME_K = MRP_K.$$

MRP_K is the **marginal revenue product** of factor K (capital). It is the

marginal physical product of capital (*i.e.*, the marginal contribution of the factor to output) multiplied with marginal revenue.

$$MRP_K = (MPP_K)(MR).$$

MRP_K is the productivity (in $) of capital. Therefore it is **the demand for capital**. Depending on the conditions in the output market, marginal revenue, MR, may or may not equal to the output price p. For example, a monopolist has MR < p. If two firms, a monopolist and a competitive firm, use the same production process and have the same physical productivity of capital MPP_K, for the same level of output the monopolist will have a lower marginal revenue product of capital, MRP_K, than the competitive firm. Since MRP_K is the demand for capital, a monopolist will demand and employ less labor than a competitive firm of the same technology.

In the input market, price-taking behavior by the buyers of inputs means than a firm can buy all the quantity it wants at a single price, v. This means that each firm that buys in the input market, sees a horizontal the supply curve for the input.

Perfectly competitive capital market
with perf. comp. or monop. in output market

$

Demand for K

$MRP_K = (MPP_K)(MR)$

v ————————————— Supply of K

$MPP_K = dq/dK$

0   K^* K

$

$(MPP_K)(MR)$

$(MPP_K)(P)$

v ————————————— Supply of K

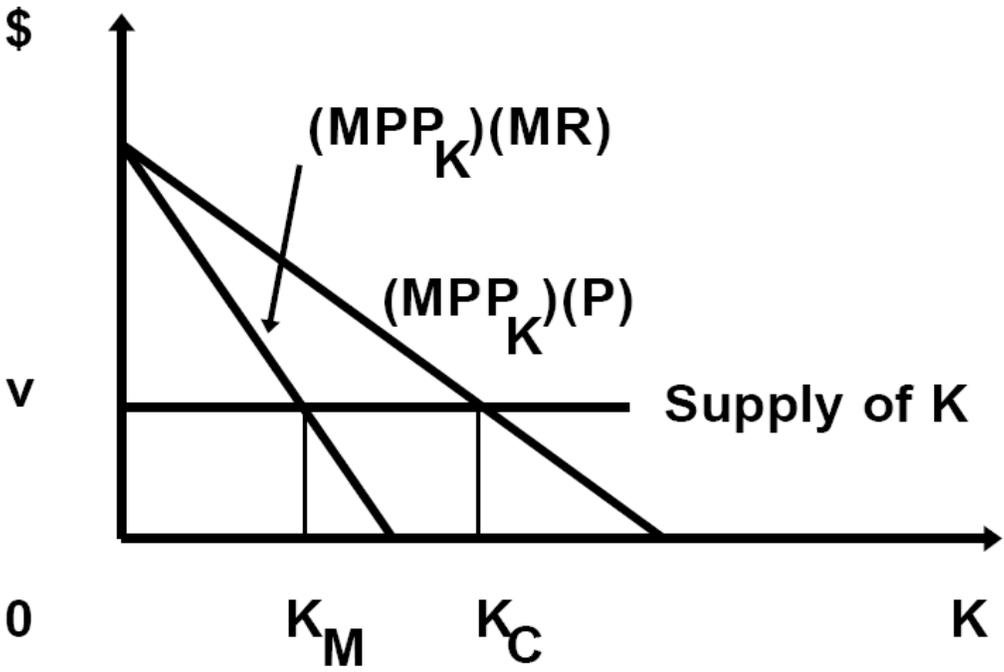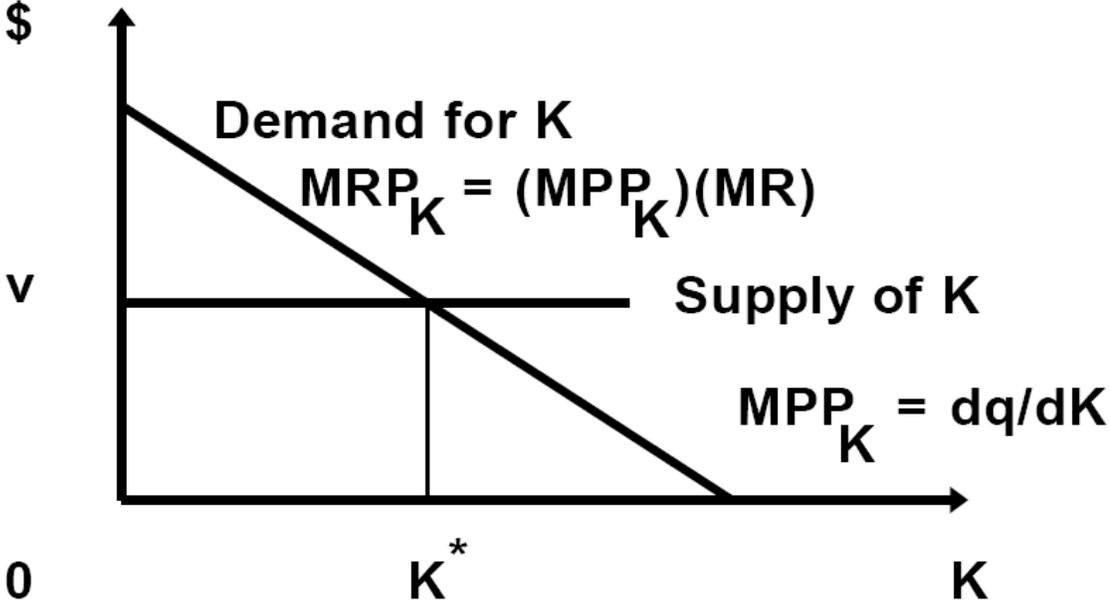0 K_M K_C K

Figure 44

59. Supply of Labor

An individual consumes **leisure** (H) and "consumption goods" (C). His utility

function is U(C, H). Both H and C are goods. Suppose that the initial

endowments are H = 24 (hours in the day) and C = C_o. The time not spent on

leisure is spend as labor L,

$$L = 24 - H.$$

Labor is a "bad" because it takes away from leisure. But, it is sold for money at

a price w (the wage rate), and money buys consumption goods C at price p

per unit. For simplicity, assume p = 1.

The budget constraint is (Figure 45)

$$C = C_o + wL, \quad i.e., \quad C = C_o + w(24 - H).$$

The slope of the budget constraint is -w, the wage rate. Maximizing utility

U(C, H) subject to the budget constraint above he chooses (C^*, H^*). This means

that he supplies $L^* = 24 - H^*$.

When the wage rate increases to w' > w, there are two effects on the consumer.

First, he has an incentive to work more, because each hour of work is worth
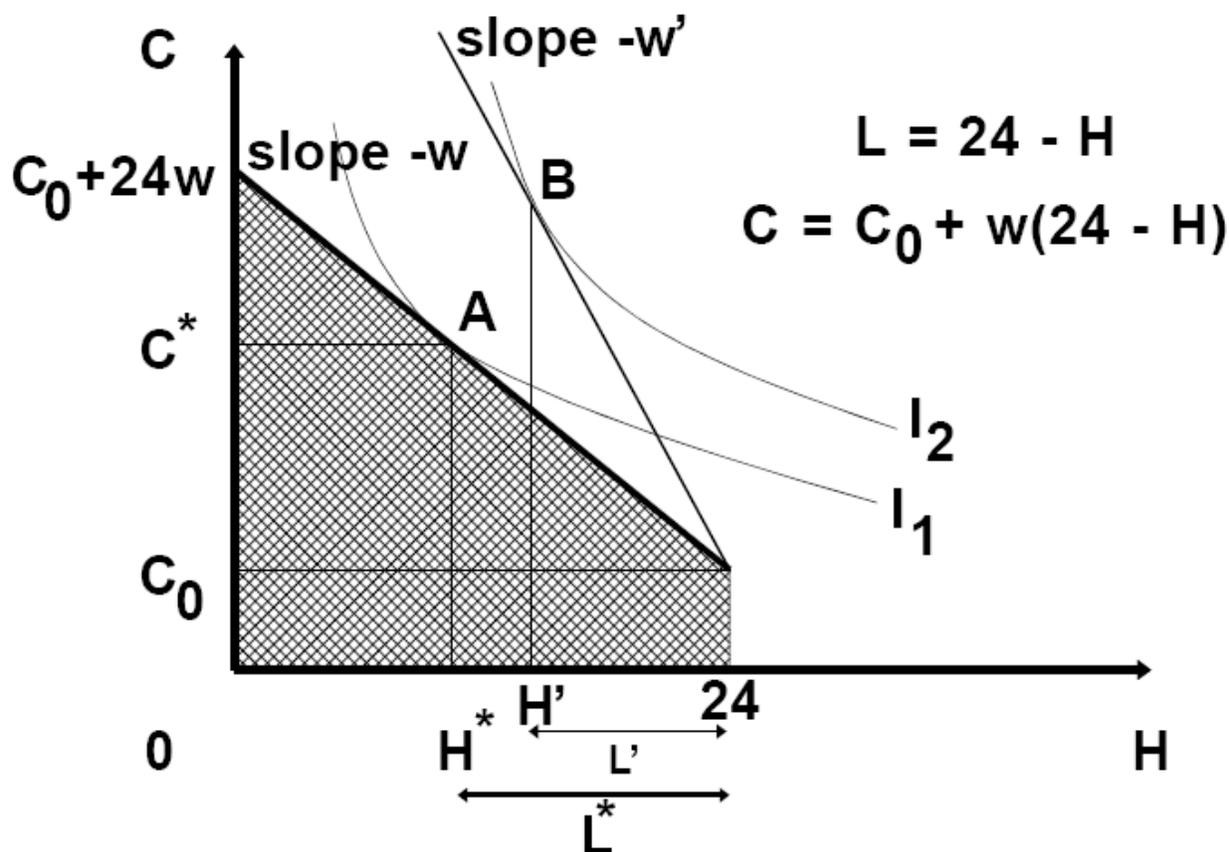
Supply of labor

Figure 45

more in consumption goods. Second, he is more rich, even if he works exactly

the same amount of hours as before. Because he is more rich, he would like to

consume more leisure and work less. Therefore the two effects of an wage

increase on the supply of labor go in opposite directions. The total effect can go

in either direction. This means that an increase in wage can lead to smaller **or**

larger supply of labor. In Figure 46, the increase in wage to w' leads to a

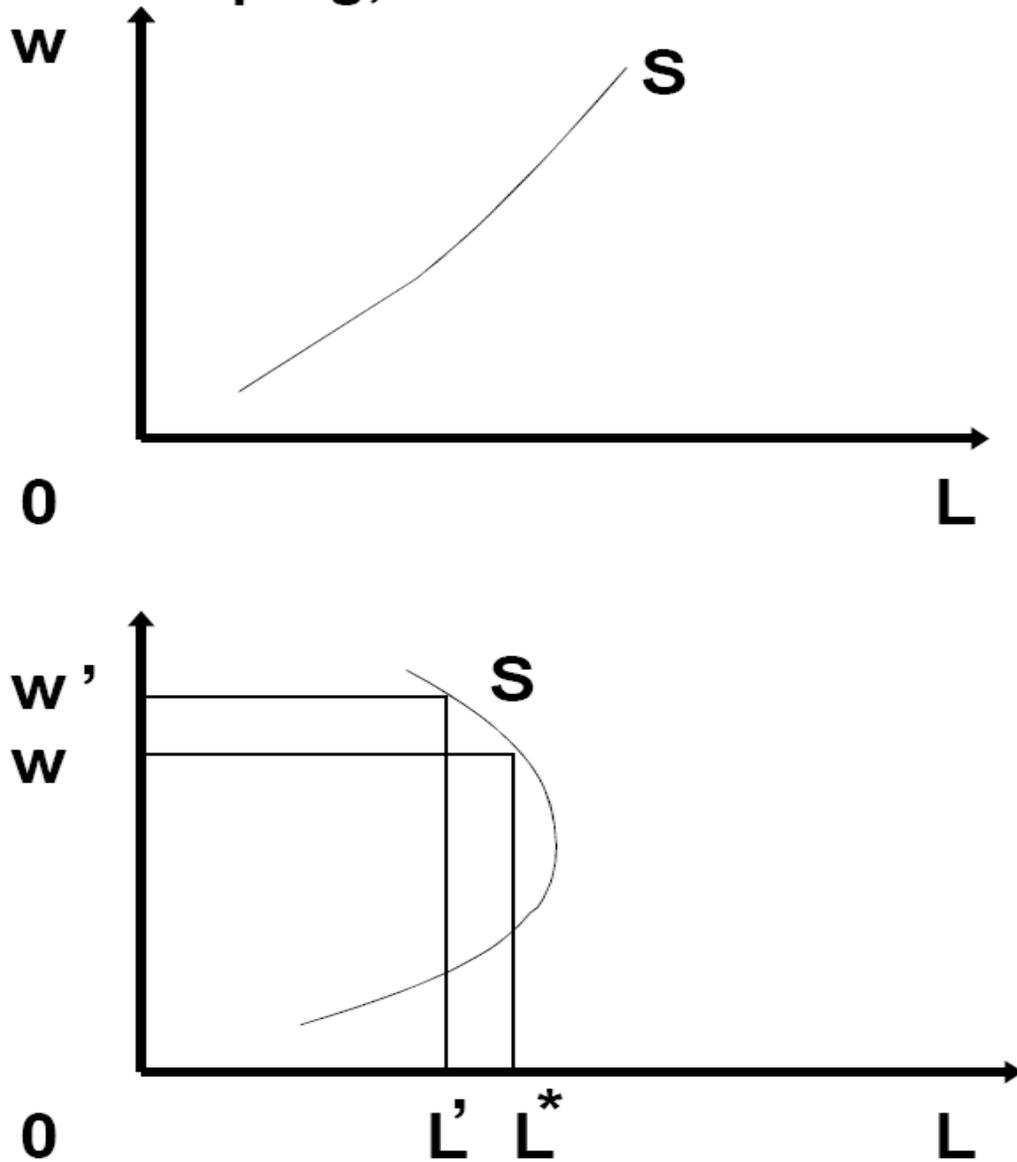## Supply of labor, upward-sloping, and backward-bending

Figure 46

reduction in the supply of labor. Such a phenomenon is called the **backward-bending supply curve** for labor. See Figure 46.

60.   **Supply of capital and intertemporal choice**. The derivation of the supply and demand for capital is a simple application of utility maximization with endowments. Suppose that there are only two periods in the world, 1 and 2 . In period 1 there is only one good c_1, and in period 2 there is only good c_2. A consumer has income m_1 in period 1 and m_2 in period 2. You should think of c_1 and m_1 as expressed in dollars of period 1; similarly, c_2 and m_2 are expressed in dollars of period 2. The consumer can consume more than m_1 in period 1 by borrowing against his income of period 2. The consumer could also consume less than m_1 in period 1 by lending some of his income. Suppose that for every dollar borrowed in period 1, the consumer has to pay $(1+r)$ dollars in period 2. Then r is called the **interest rate**. Suppose that the consumer can borrow and lend at the same rate. If the consumer borrows $(c_1 - m_1)$ in period 1 he has to pay back $(c_1 - m_1)(1 + r)$ in period 2. Since there is no later period, this amount has to equal the difference between income and consumption in period 2, *i.e.*,

$$m_2 - c_2 = (c_1 - m_1)(1 + r), \quad i.e., \quad c_2 = m_2 + (m_1 - c_1)(1 + r).$$

This is the budget constraint of the consumer. See Figure 47. The consumer's preferences between consumption in period 1 and consumption in period 2 are expressed through his indifference curves. Maximizing satisfaction, the

consumer picks point A where there is tangency between the indifference curve

and the budget constraint. In Figure 47, the consumer spends c_1^* in period 1,

and borrows $c_1^* - m_1$. He spends only c_2^* in period 2. He uses the remaining of

his income in period 2, $m_2 - c_2^*$ to pay his debt, which has grown with interest to

$(c_1^* - m_1)(1 + r)$. We know from the budget constraint that these two amounts

are equal.

Supply of Capital

budget constraint:
$$m_2 - c_2 = (c_1 - m_1)(1+r)$$

slope $-(1+r)$

Endowment

Borrowing $= c_1^* - m_1$

c_2 $m_2 + m_1(1+r)$ m_2 E   A c_2^*

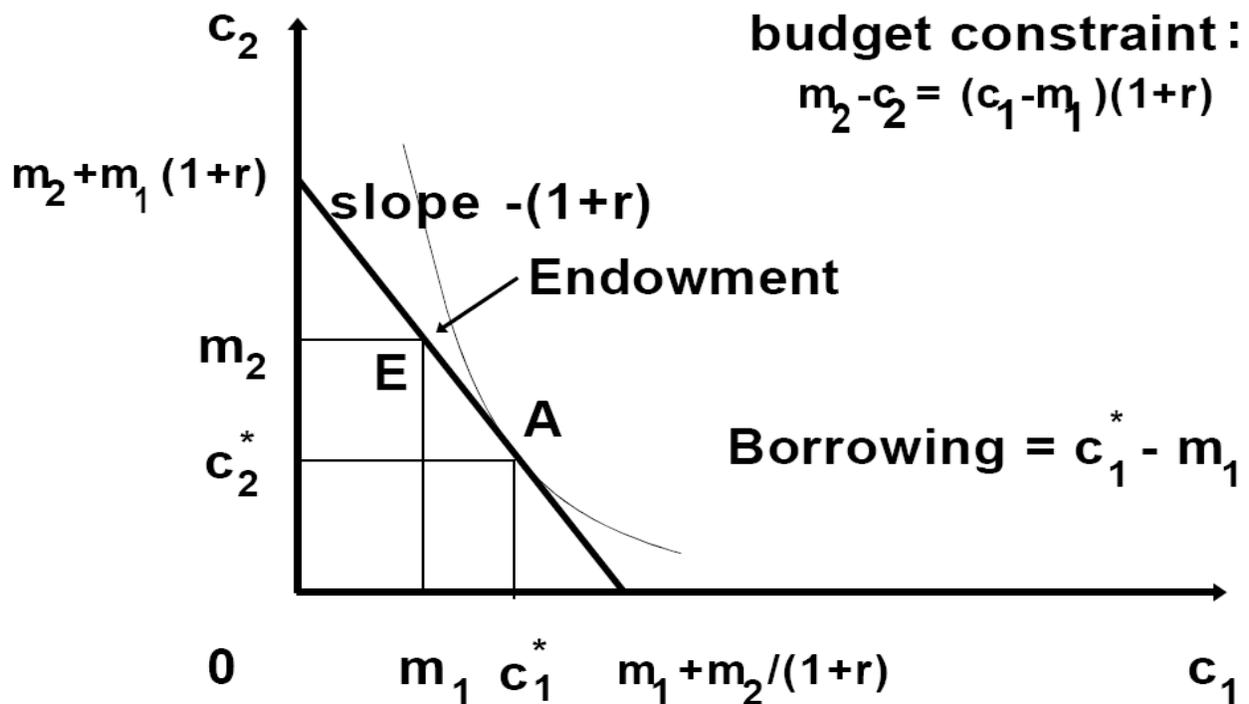0 m_1 c_1^* $m_1 + m_2/(1+r)$ c_1

Figure 47

61.   **Interest rates**. As we have seen, given the interest rate, some consumers

decide to borrow and some consumers decide to lend money. Summing the

amounts the consumers are willing to borrow in period 1 at different interest

rates we derive the **demand** for funds in period 1. Similarly, summing the

amounts the consumers are willing to lend in period 1 at different interest rates

we derive the **supply** of funds in period 1. The intersection of supply and

demand determines the market interest rate.

62.   **Present value**. A project **x** can be thought of as a stream of incomes in

consecutive time periods $\mathbf{x} = (x_0, x_1, ..., x_n)$. Any of these numbers can be zero,

positive or negative. An easy way to evaluate projects is by computing present

values. The present value of next year's return is $x_1/(1+r_1)$, where r_1 is the

interest rate between periods 0 and 1. The present value in period 0 of period

2's return is $x_2/[(1+r_1)(1+r_2)]$, where r_2 is the interest rate between period 1 and

2. Similarly, the present value in period 0 of period n's return is

$x_n/[(1+r_1)(1+r_2)...(1+r_n)]$, where r_n is the interest rate between period n-1 and n.

Note that in general the interest rate between period 0 and 1 will be different

than the interest rate between 1 and 2, and so on. For the project $\mathbf{x} = (x_0, x_1, ...,$

$x_n)$ that has returns in all periods, the **present value (PV)** is defined as

$$PV(\mathbf{x}) = x_0 + x_1/(1+r_1) + x_2/[(1+r_1)(1+r_2)] + ... + x_n/[(1+r_1)...(1+r_n)].$$

In adopting projects, a manager can use present values, and adopt the project

that has the higher PV.

63.   **Bonds and relationship to interest rates**. A **bond** is an obligation by the US government or other entities. Depending on the issuer, bonds have different risks. The risk of a bond is the risk of default and no final payment of the obligation that the bond represents. The least risky bonds are the ones issued by the US Treasury. Their low risk is reflected in their low return. High risk bonds (junk bonds) have higher returns.

64.   We now consider the determination of the price of bonds and their relationship with interest rates. This discussion is limited to riskless (US Treasury) bonds. A typical US Treasury Bond (or Note) **B** has a **face value** F, and a **coupon** C. A bond with expiration after n periods pays F after n periods; it also pays C at the end of each period between now and its expiration. From the point of view of an individual that buys it, the bond can be considered as a "project". The stream of returns for the bond are (C, C, ..., C, F), where C appears in the first n-1 positions, and F in the nth position. What is the present value of the bond? Using the formula we have,

$$PV(\mathbf{B}) = C + C/(1+r_1) + C/[(1+r_1)(1+r_2)] + ... + C/[(1+r_1)...(1+r_{n-1})]$$

$$+ F/[(1+r_1)...(1+r_n)].$$

Note that the present value of a bond is inversely related to the interest rates r_1, r_2, ..., r_n.

65. The US Treasury issues bonds and notes of many durations. For simplicity, suppose a period is a year, and there are bonds of duration of one year $\mathbf{B_1}$, two years $\mathbf{B_2}$, and so on to n years $\mathbf{B_n}$. Assume also that irrespective of length, all bonds have the same face value F and same coupon C. Their present values are easy to calculate using the general formula. For example, the present value of the one-year bond is,

$$PV(\mathbf{B_1}) = C + F/(1 + r_1).$$

Now consider that these bonds are auctioned today. They must fetch exactly their present value. Given the interest rate r_1, there is a unique present value for bond $\mathbf{B_1}$. Conversely, if we know the price that the bond fetched in the auction (and which is its present value) we can use the formula above to find the interest rate r_1,

$$PV(\mathbf{B_1}) = C + F/(1 + r_1) \Leftrightarrow r_1 = F/[PV(\mathbf{B_1}) - C] - 1.$$

In this way, *the **auction** of one-year bonds reveals the one-year interest rate*. Now consider the two-year bond $\mathbf{B_2}$. Its present value is

$$PV(\mathbf{B_2}) = C + C/(1+r_1) + F/[(1+r_1)(1+r_2)].$$

When it is auctioned today, its price is PV($\mathbf{B}_2$). Thus, we know the value of the Left-Hand-Side of the formula. In the Right-Hand-Side of this formula, we already know r_1 from the analysis of the one-year bond. The value of the interest rate r_2 follows immediately. We can repeat this analysis step by step for bonds of longer duration. In this way, the auction prices of US treasury bonds reveal the interest rates of varying durations.

66.   Application to the EU financial crisis: bonds exchange in the face of uncertainty when participants have varying expectations. Consider the secondary market for Greek sovereign bonds that mature a year from now that have a coupon C and face value F (which are known to all), and suppose that, assuming no default, the interest rate for Greek bonds is r_1. For example, assume that $r_1 = 5\% = 0.05$. Once we take into account the expectations of default by market participants, the market interest rate will be different than r_1. We will construct the demand for the Greek bond and find the market interest rate. To make things concrete, suppose there are €40 billion of such bonds in circulation (total supply), a vertical line at 40 billion.

67.   Suppose we are absolutely sure that Greece will pay the coupon C, but there is uncertainty about whether Greece will pay the full face value after a

year. If Greece pays the full face value after a year, the bond present value is

$PV_1 = C + F/(1 + r_1)$. If everyone believes that Greece will pay only x (as

fraction of 1) of the bond's face value (*i.e.*, it will impose a "haircut" of (1-x) of

the bond's value), the bond present value is $PV(x) = C + xF/(1 + r_1)$. This is the

same present value as of a bond of interest rate r_x and certainty of payment:

$PV(x) = C + F/(1 + r_x)$ provided that $r_x = (1- x + r_1)x = (1 + r_1)/x - 1$, which

comes from $xF/(1 + r_1) = F/(1 + r_x)$.

% Of Face Value Paid	Haircut As %	Certainty Interest Rate	Uncertainty Implied Interest Rate
x	1-x	r1	rx = (1 + r1)/x − 1
100	0	5.00%	5.00%
95	5	5.00%	10.53%
90	10	5.00%	16.67%
85	15	5.00%	23.53%
80	20	5.00%	31.25%
75	25	5.00%	40.00%
70	30	5.00%	50.00%

Even a small haircut implies a high interest rate (discount rate). A 10% haircut

results in a 16.7% interest rate (from 5% interest rate without a haircut). A 20%

haircut implies a 31.25% interest rate (close to the present interest rate for

Greek bonds). But if we observe a high implied interest rate, does this mean that

everyone believes there will be a haircut?

68.   Suppose investors are divided on their expectations of whether Greece

will impose a haircut. Example: half of the investors expect no haircut and the

other half expects a 20% haircut. This implies that the demand curve consists of

two lines: a horizontal line at price PV(1) for quantities 0 to 20 billion, and a

second horizontal at price PV(0.8) for quantities 20 to 40 billion . You can

easily see that the equilibrium price for the bond is PV(0.8), which brings the

discount rate from 5% to 31.25%. This means that it is sufficient for some part

of the market to expect a haircut for the interest rate to soar.

F.   GAINS FROM TRADE AND COMPETITIVE EQUILIBRIUM

69.   **Gains from trade**. Suppose there are two goods, X, Y, and two

individuals (traders), S, and J, endowed with (x_A^S, y_A^S) and (x_A^J, y_A^J) respectively.

We construct the **Edgeworth box** of dimensions $x_A^J + x_A^S$ and $y_A^J + y_A^S$. A

point, such as A, in the Edgeworth box is an **allocation**. The traders J and S

can reach any other point, such as B, by trading an amount of X for an amount

of Y. See Figure 48.

70.   We can draw the indifference curves for person J and person S through

point A. In general, these indifference curves intersect (*i.e.*, are not tangent),

and therefore they create a **lens** of **Pareto superior** allocations to A. Any point,

such as C, in the lens is preferred to A by both persons. Starting from A, both
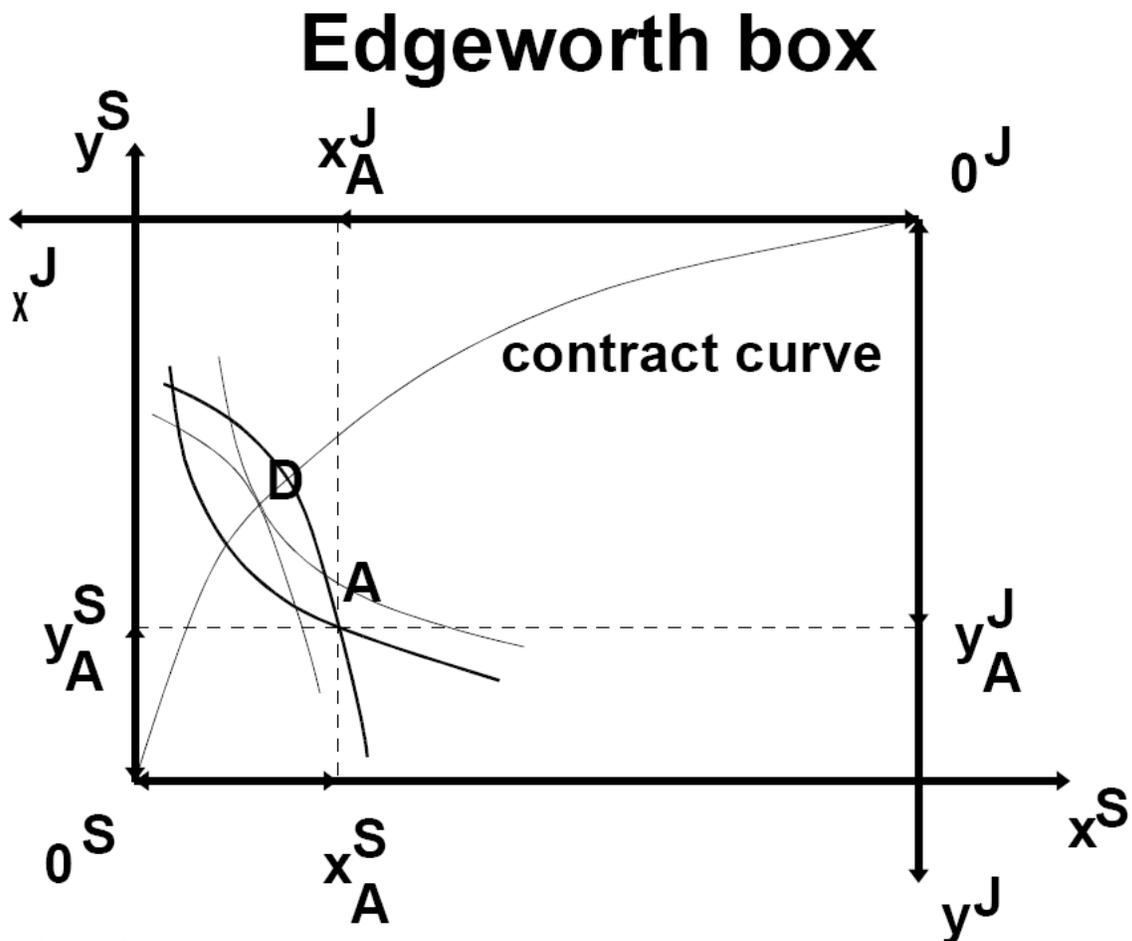
persons are willing to trade to reach C.

Edgeworth box

Figure 48

71.   A point, such as D, where the two indifference curves passing through it

are tangent, cannot be improved upon through trade. There is no lens to move

to. Such a point is called **Pareto optimal**. The collection of Pareto optimal

points is called the **contract curve**. The portion of the contract curve in the

lens is called the **core**. It consists of all allocations that have two properties. (1)

 They are superior to A for both persons (and therefore are in the lens); and (2)

They are Pareto optimal (and therefore are on the contract curve).

Budget constraint
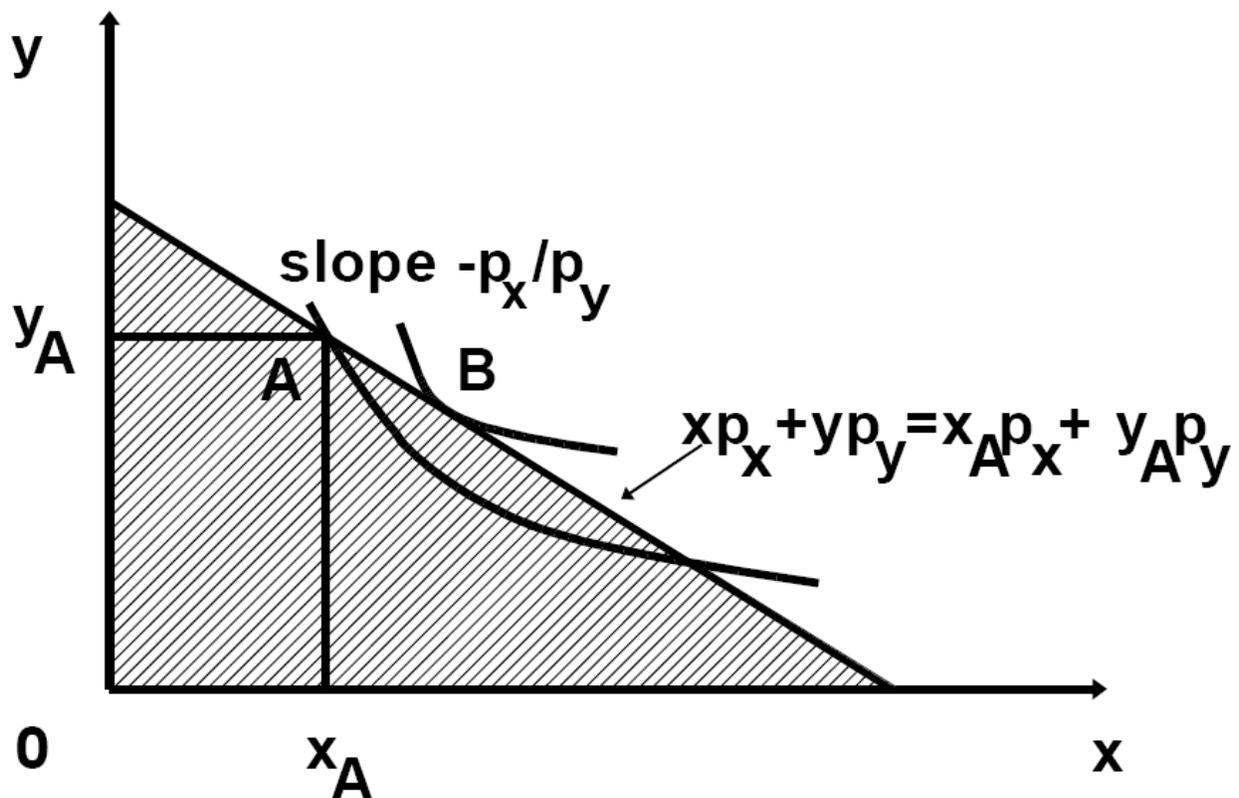## and optimal point with endowments

Figure 49

72.   One common way to trade is with constant prices. An individual with

endowment (x_A, y_A) has budget constraint

$$p_x x + p_y y = p_x x_A + p_y y_A.$$

The right-hand-side is the value of her endowment. The left-hand side is the value of the consumed bundle (x, y). In Figure 49, the consumer chooses bundle B which has less y and more x. Therefore, starting with bundle A, the consumer sells y and buys x to reach bundle B.

73.   A **competitive equilibrium** has the following properties: (1) Each trader maximizes utility for given prices p_x, p_y; (2) The total quantity demanded by consumers is equal to the total supply.

In an Edgeworth box, the budget line *for both traders* passes through the point of initial endowments, A, and has slope -p_x/p_y. Suppose that both traders maximize their utility by picking up the *same* point, B, on the budget line. See Figure 51. We know that *any* point in the Edgeworth box signifies a feasible distribution of total resources. In other words, the total quantity demanded of each good at B is equal to the total resources,

$$x_B^J + x_B^S = x_A^J + x_A^S \text{ and } y_B^J + y_B^S = y_A^J + y_A^S.$$

Therefore, *if traders pick up the **same** point, B, then we have a competitive equilibrium.*

The significance of the competitive equilibrium is that traders can reach a
Pareto optimal point without meeting or bargaining. Traders just observe
prices. If all markets clear, a Pareto optimal point has been reached. In the
diagram, for markets to clear, both traders must reach the same point B.
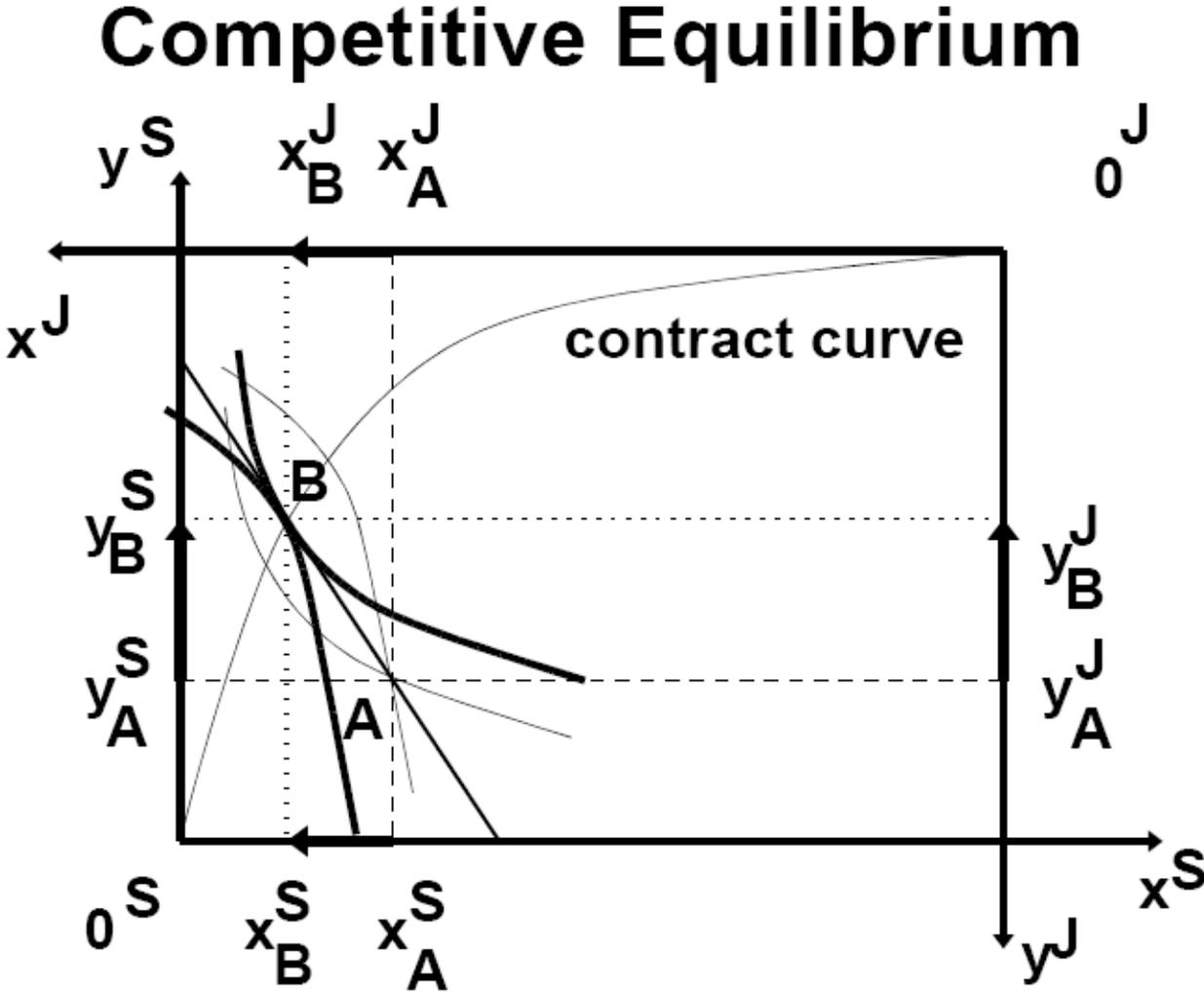
Competitive Equilibrium

Figure 50

Because B is optimal for trader S, his indifference curve is tangent to the budget constraint. Similarly because B is optimal for trader J, his indifference curve is tangent to the budget constraint. Noticing that the budget constraint is the same line because it passes through A and it has slope $-p_x/p_y$, it follows that the two indifference curves (for S and J) are tangent to the same line and therefore tangent to each other. Since the two indifference curves are tangent to each other at B, there is no possibility of mutual improvement starting from B (no lens). Therefore B is Pareto optimal.

74. **Disequilibrium**. For different prices, consumers may pick different points. For example, in Figure 51, the budget line is drawn with a different slope. Sandy picks B and Jamie picks C. The arrows indicate the trades that Jamie and Sandy are willing to make. Sandy is willing to give up more x (lower horizontal arrow) than Jamie is willing to buy (upper horizontal arrow). Therefore there will be leftover x after trade. But Sandy wants more y (left vertical arrow) than Jamie is willing to give up (right vertical arrow).
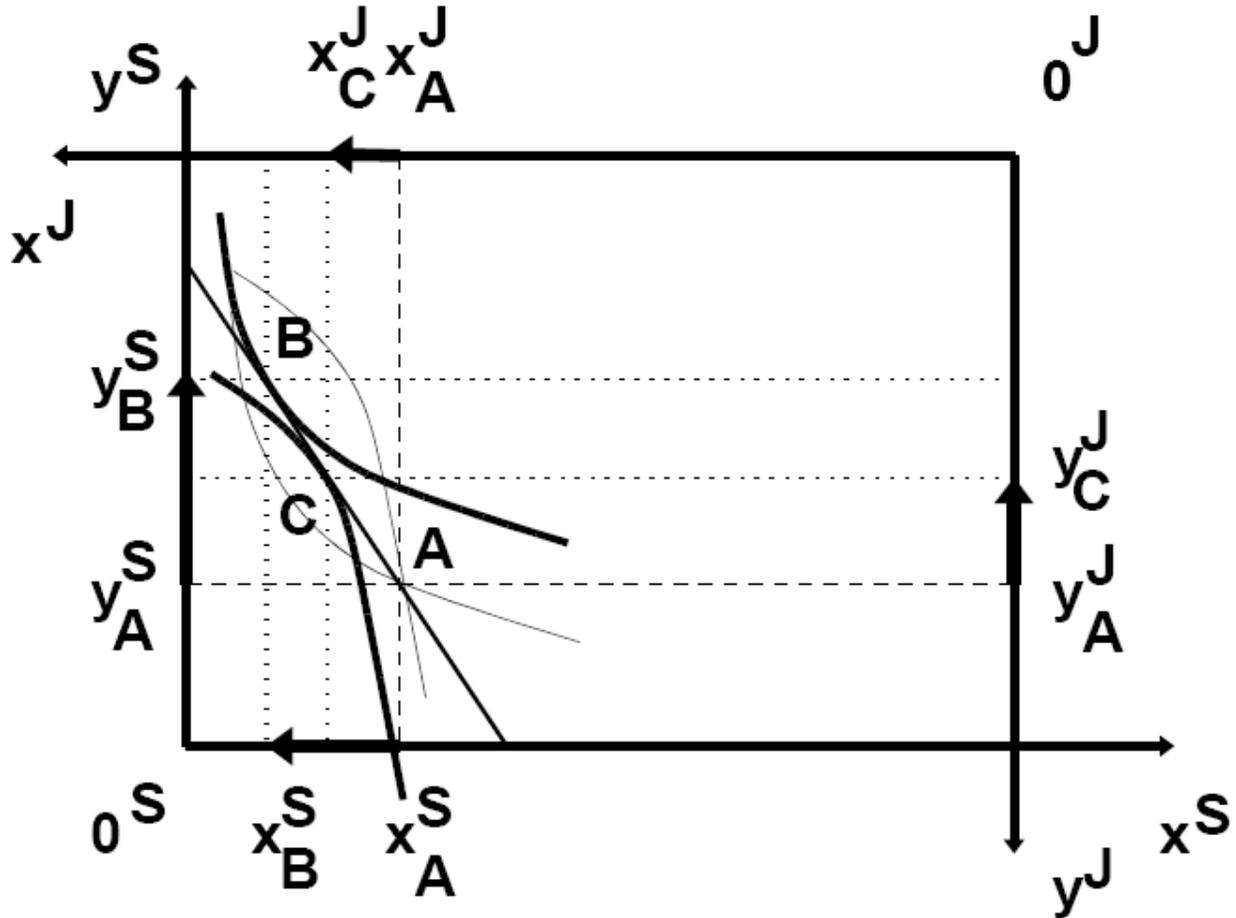
Disequilibrium

Figure 51

Therefore there will be more demand for y than the total supply available.

$$\text{demand of } y = y_C^J + y_B^S > y_A^J + y_A^S = \text{supply of } y$$

This is a **disequilibrium** situation. In Figure 52, the excess demand for y is

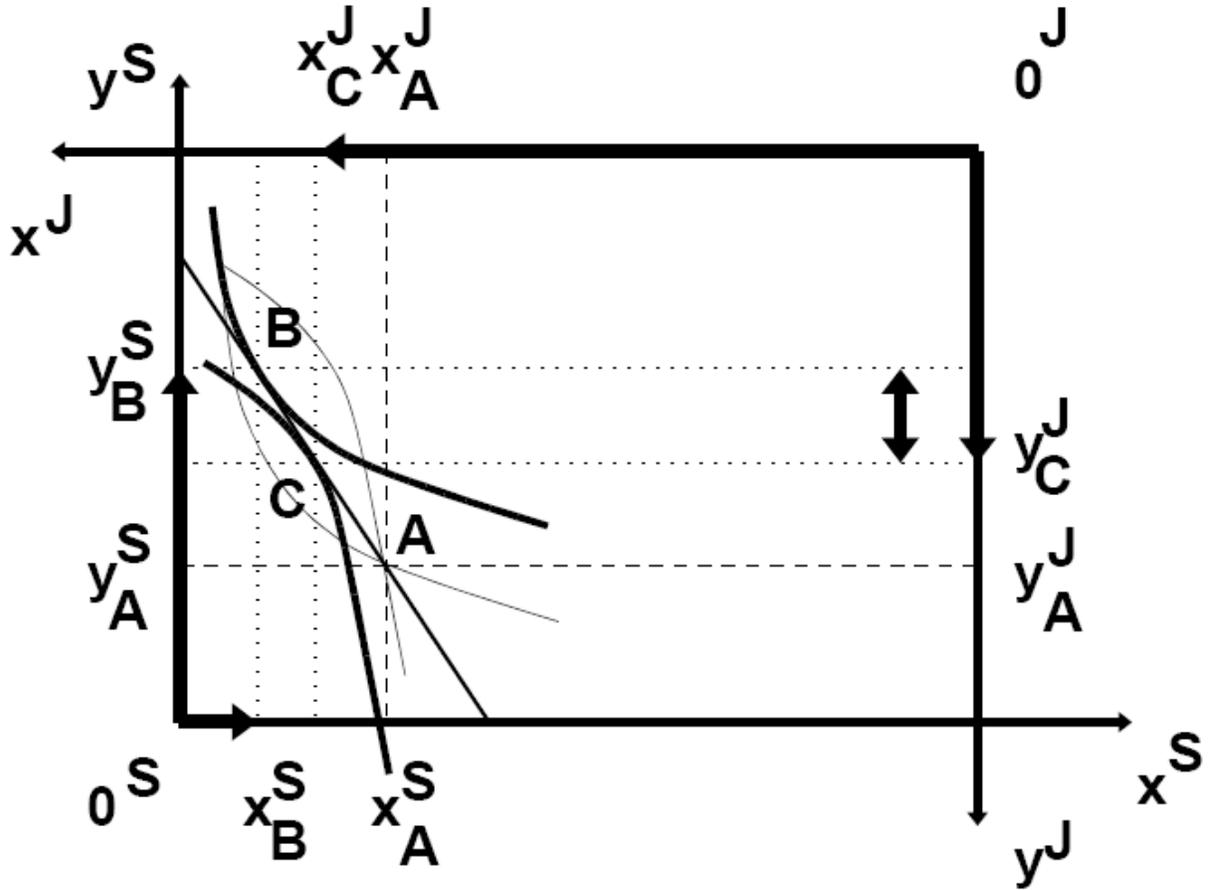seen as a vertical double arrow on the far right.

99

Disequilibrium

Figure 52

G.   NETWORKS AND THE "NEW ECONOMY"

To be distributed separately.